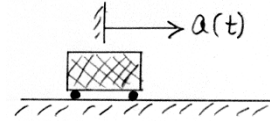


ME 2580 Example #2: (Rectilinear Motion)

Given: $a(t) = 10 - 2t^2$ (m/s²) ... the **acceleration** of some particle
initial conditions: $v(0) = 10$ (m/s) and $s(0) = 5$ (m)



Find: a) $v(t)$... the **velocity** of the particle **as a function of time**
 b) $s(t)$... the **position** of the particle **as a function of time**
 c) s_T ... the **total distance traveled** from $0 \rightarrow 10$ (sec)

Solution:

a) $\frac{dv}{dt} = a(t) \Rightarrow \int_{v(0)}^{v(t)} dv = \int_0^t a(t) dt \Rightarrow v(t) - v(0) = \int_0^t (10 - 2t^2) dt$ using definite integrals

$\Rightarrow v(t) = v(0) + \left(10t - \frac{2}{3}t^3\right)_0^t \Rightarrow v(t) = 10 + 10t - \frac{2}{3}t^3$ (m/s)

or

$v(t) = \int a(t) dt = \int (10 - 2t^2) dt = 10t - \frac{2}{3}t^3 + D$ using indefinite integrals

then, using the **initial condition**

$v(0) = 10 = \left(10t - \frac{2}{3}t^3 + D\right)_{t=0} = D \Rightarrow v(t) = 10 + 10t - \frac{2}{3}t^3$ (m/s)

b) $s(t) = \int v(t) dt = \int \left(10 + 10t - \frac{2}{3}t^3\right) dt = 10t + 5t^2 - \frac{1}{6}t^4 + D$ using indefinite integrals

then, using the **initial condition**

$s(0) = 5 = \left(10t + 5t^2 - \frac{1}{6}t^4 + D\right)_{t=0} = D \Rightarrow s(t) = 5 + 10t + 5t^2 - \frac{1}{6}t^4$ (m)

c) To find the **total distance traveled** from $0 \rightarrow 10$ (sec), we need to determine if the particle **changes direction** during that time. To do this, we find the times when the **velocity is zero**.

$v(t) = 10 + 10t - \frac{2}{3}t^3 = 0 \Rightarrow t = \begin{cases} -3.215 \times \\ -1.085 \times \\ 4.2998 \end{cases}$

So, we can calculate the **total distance** traveled by first calculating the locations of the particle at the beginning and end of the interval and when the particle changes direction:

$s(0) = 5$, $s(4.2998) = 83.47$, and $s(10) = -1061.67$

$s_T = \underbrace{(83.47 - 5)}_{\text{distance from starting position to zero velocity}} + \underbrace{83.47}_{\text{distance to return to origin}} + \underbrace{1061.67}_{\text{distance from origin to final position}} = 1223.61 \approx 1220$ (m)