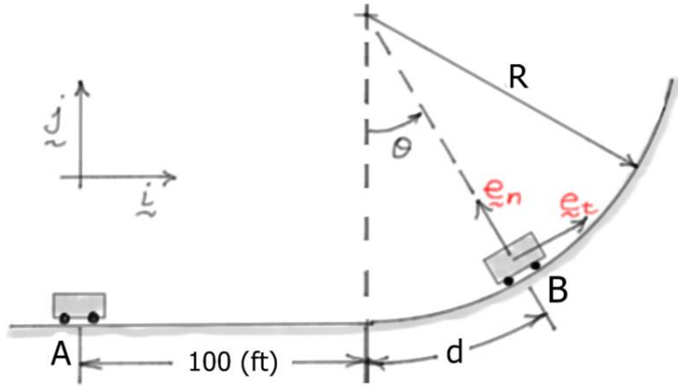
ME 2580 Example #7: (2D Motion, Normal & Tangential Components)

Given: $R = 200$ (ft), $d = 12$ (ft)

car starts from rest at A

car accelerates at a constant rate of

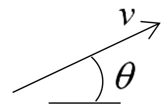
$\dot{v} = a_t = 10$ (ft/s^2) from A to B



Find: $\underset{\sim}{v}_B$ and $\underset{\sim}{a}_B$ in ft/s and ft/s^2 using **_normal_** and **_tangential_** components.

Solution:

Velocity: (given constant acceleration along the path)



$$v_B = \sqrt{v_A^2 + 2a_t\,\Delta s} = \sqrt{2(10)(112)} \approx 47.3286 \approx 47.\overline{3} \text{ (ft/s)} \quad \Rightarrow \quad \underset{\sim}{v}_B = 47.3\,\underset{\sim}{e}_t \text{ (ft/s)}$$

Acceleration:

$$a_t = \dot{v} = 10 \text{ (ft/s}^2\text{)} \qquad a_n = \frac{v^2}{\rho} = \frac{2(10)(112)}{200} = 11.2 \text{ (ft/s}^2\text{)}$$

$$\underset{\sim}{a}_B = 10\,\underset{\sim}{e}_t + 11.2\,\underset{\sim}{e}_n \text{ (ft/s}^2\text{)} \quad \Rightarrow \quad |\underset{\sim}{a}_B| = \sqrt{a_t^2 + a_n^2} \approx 15.0147 \approx 15.0 \text{ (ft/s}^2\text{)}$$

Aside:

We could also express these results in terms of the unit vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$. For example,

$$\theta = d/R = (12/200)(180/\pi) \approx 3.43775 \approx 3.44 \text{ (deg)}$$

$$\underset{\sim}{v}_B = 47.3\,\underset{\sim}{e}_t = 47.3\left(\cos(\theta)\,\underset{\sim}{i} + \sin(\theta)\,\underset{\sim}{j}\right) = 47.2\,\underset{\sim}{i} + 2.84\,\underset{\sim}{j} \text{ (ft/s)}$$

The acceleration could also be expressed in terms of the unit vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$ by noting that: $\underset{\sim}{e}_n = -\sin(\theta)\,\underset{\sim}{i} + \cos(\theta)\,\underset{\sim}{j}$.