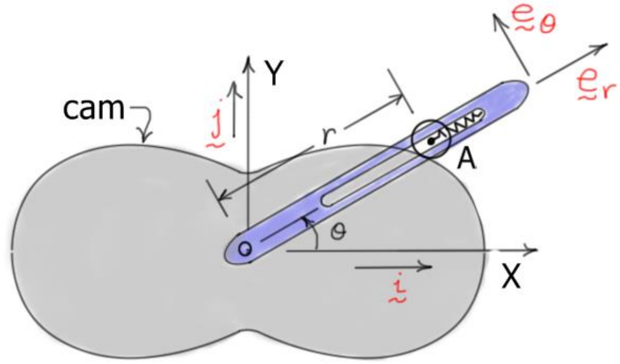
ME 2580 Example #10: (2D Motion, Radial & Transverse Components)



Given: $\dot{\theta} = \omega = 10$ (rad/s) … constant

$r(\theta) = 1 + 0.5\cos(2\theta)$ (ft)

Find: $\underset{\sim}{v}_A$ and $\underset{\sim}{a}_A$ in ft/s and ft/s^2 using ***radial*** and ***transverse*** components at $\theta = 30$ (deg) .

Solution:

Velocity:

$$\underset{\sim}{v} = v_r\, \underset{\sim}{e}_r + v_\theta\, \underset{\sim}{e}_\theta$$

$$v_r = \frac{dr}{dt} = \frac{d}{dt}\big(1 + 0.5\cos(2\theta)\big) = 0.5\big(-\sin(2\theta)\big)\big(2\dot{\theta}\big)$$

chain rule

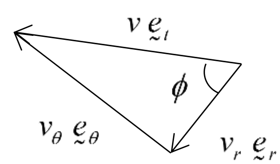
$$= -\dot{\theta}\sin(2\theta) \approx -8.66025 \approx -8.66 \text{ (ft/s)}$$

$$v_\theta = r\dot{\theta} = \big(1 + 0.5\cos(2\theta)\big)(10) = (1.25)(10) = 12.5 \text{ (ft/s)}$$

$$\underset{\sim}{v}_A = -8.66\, \underset{\sim}{e}_r + 12.5\, \underset{\sim}{e}_\theta \text{ (ft/s)}$$

$$|\underset{\sim}{v}_A| = \sqrt{v_r^2 + v_\theta^2} \approx 15.2069 \approx 15.2 \text{ (ft/s)}$$ (tangent to the cam surface)

$$\phi = \tan^{-1}\left(\frac{v_\theta}{|v_r|}\right) = \tan^{-1}\left(\frac{12.5}{8.66}\right) \approx 55.3 \text{ (deg)}$$



Acceleration:

$$\underset{\sim}{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\underset{\sim}{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\underset{\sim}{e}_\theta$$

$$\ddot{r} = \frac{d\dot{r}}{dt} = \frac{d}{dt}\big(-\dot{\theta}\sin(2\theta)\big) = -\underbrace{\ddot{\theta}\sin(2\theta)}_{\text{zero}} - \dot{\theta}\big(\cos(2\theta)\big)\big(2\dot{\theta}\big) = -2\dot{\theta}^2\cos(2\theta)$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2\dot{\theta}^2\cos(2\theta) - 1.25\left(10^2\right) = -225 \text{ (ft/s}^2\text{)}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + \big(2(-8.66025)(10)\big) = -173.205 \approx -173 \text{ (ft/s}^2\text{)}$$

$$\underset{\sim}{a}_A = -225\, \underset{\sim}{e}_r - 173\, \underset{\sim}{e}_\theta \text{ (ft/s}^2\text{)}$$