

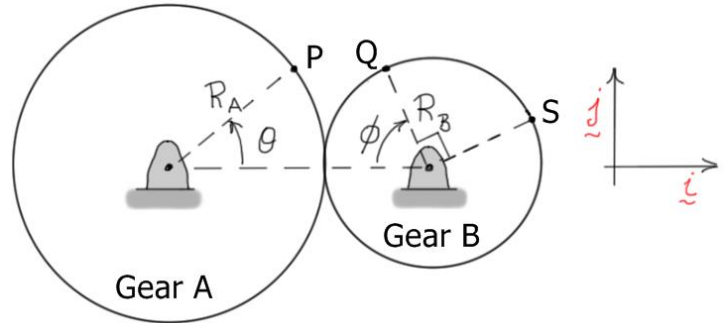
ME 2580 Example #12: (2D Motion, Relative Motion)

Given: $R_A = 3$ (in), $R_B = 2$ (in)

$\omega_A = \dot{\theta} = 1000$ (rpm) ... constant

$\omega_B = \dot{\phi} = 1500$ (rpm) ... constant

➤ when $\theta = \phi = 0$, P and Q are touching, and S is directly above the support.



Find: $\underline{v}_{Q/P}$, $\underline{a}_{Q/P}$, $\underline{v}_{S/P}$, $\underline{a}_{S/P}$ at $\theta = \phi = 0$

Solution:

$$\underline{v}_P = R_A \dot{\theta} \underline{j} = \left(\frac{3}{12}\right) \left(1000 \left(\frac{2\pi}{60}\right)\right) \underline{j} \approx 26.1799 \underline{j} \approx 26.2 \underline{j} \text{ (ft/s)}$$

$$\underline{v}_Q = R_B \dot{\phi} \underline{j} = \left(\frac{2}{12}\right) \left(1500 \left(\frac{2\pi}{60}\right)\right) \underline{j} \approx 26.1799 \underline{j} \approx 26.2 \underline{j} \text{ (ft/s)}$$

$$\underline{v}_{Q/P} = \underline{v}_Q - \underline{v}_P = \underline{0}$$

Note: for gears in contact, $R_A \dot{\theta} = R_B \dot{\phi}$ where R_A and R_B represent the *pitch radii* of the two gears.

$$\underline{a}_P = -\left(R_A \dot{\theta}^2\right) \underline{i} = -\left(\frac{3}{12}\right) \left(1000 \left(\frac{2\pi}{60}\right)\right)^2 \underline{i} \approx -2741.56 \underline{i} \text{ (ft/s}^2\text{)}$$

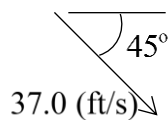
$$\underline{a}_Q = \left(R_B \dot{\phi}^2\right) \underline{i} = \left(\frac{2}{12}\right) \left(1500 \left(\frac{2\pi}{60}\right)\right)^2 \underline{i} \approx 4112.34 \underline{i} \text{ (ft/s}^2\text{)}$$

$$\underline{a}_{Q/P} = \underline{a}_Q - \underline{a}_P \approx 6850 \underline{i} \text{ (ft/s}^2\text{)}$$

So, the *relative acceleration* of the two contacting points is **not zero**.

$$\underline{v}_S = R_B \dot{\phi} \underline{i} = \left(\frac{2}{12}\right) \left(1500 \left(\frac{2\pi}{60}\right)\right) \underline{i} \approx 26.1799 \underline{i} \approx 26.2 \underline{i} \text{ (ft/s)}$$

$$\underline{v}_{S/P} = \underline{v}_S - \underline{v}_P = 26.2 \underline{i} - 26.2 \underline{j} \text{ (ft/s)}$$



Finally,

$$\underline{a}_S = -\left(R_B \dot{\phi}^2\right) \underline{j} = -\left(\frac{2}{12}\right) \left(1500 \left(\frac{2\pi}{60}\right)\right)^2 \underline{j} \approx -4112.34 \underline{j} \text{ (ft/s}^2\text{)}$$

$$\underline{a}_{S/P} = \underline{a}_S - \underline{a}_P \approx 2740 \underline{i} - 4110 \underline{j} \text{ (ft/s}^2\text{)}$$

