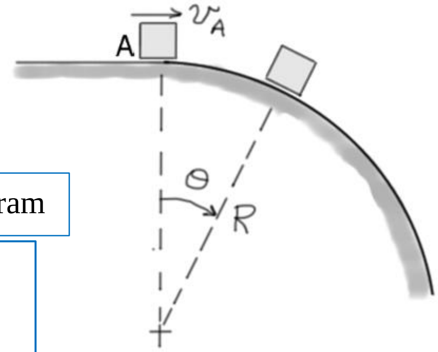
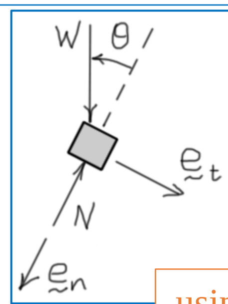
ME 2580 Example #16: (Newton's Laws, Normal & Tangential Components)



Given: $W_{\text{box}} = W = 5$ (lb), $R = 2$ (ft), $v_A = 4$ (ft/s)
surface is smooth (no friction)

Find: $\hat{\theta}$ the angle where the box begins to leave the surface

Free body diagram



Solution:

$$\swarrow \sum F_n = W\cos(\theta) - N = \left(\tfrac{W}{g}\right)\left(\tfrac{v^2}{R}\right)$$

$$\searrow \sum F_t = W\sin(\theta) = \left(\tfrac{W}{g}\right)\dot{v}$$

Using the second equation,

using indefinite integrals

$$\frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt} = \frac{v}{R}\frac{dv}{d\theta} = g\sin(\theta) \Rightarrow \int v\,dv = \int g\,R\sin(\theta)\,d\theta \Rightarrow \tfrac{1}{2}v^2 = -gR\cos(\theta) + D$$

Initial condition: @ $\theta = 0$, $v = v_A = 4$ (ft/s) $\Rightarrow$ $D = \tfrac{1}{2}4^2 + gR\cos(0) = 8 + 2g$

$$\Rightarrow v^2 = 2\big(8 + 2g - 2g\cos(\theta)\big) \qquad (R = 2 \text{ (ft)})$$

When the box begins to leave the surface, $N = 0$:

$$\sum F_n = W\cos(\hat{\theta}) - N = W\cos(\hat{\theta}) = \left(\tfrac{W}{g}\right)\left(\tfrac{v^2}{R}\right) = \left(\tfrac{W}{g}\right)\left(\tfrac{v^2}{2}\right) = \left(\tfrac{W}{g}\right)\big(8 + 2g - 2g\cos(\hat{\theta})\big)$$

$$\Rightarrow (3W)\cos(\hat{\theta}) = \left(\tfrac{W}{g}\right)(8 + 2g) \Rightarrow \hat{\theta} = \cos^{-1}\left(\frac{8 + 2g}{3g}\right) = 41.454 \approx 41.5 \text{ (deg)}$$