

ME 2580 Example #20: (Work & Energy)

Given: $W_A = 100$ (lb), $W_B = 110$ (lb), $\mu_k = 0.3$, $\theta = 20$ (deg)
system is **released from rest** and B moves down

Find: velocities of A and B after moving a distance
 $d = 2$ (ft)

Solution:

Newton's 2nd Law:

$$A: \sum F_y = N - W_A \cos(\theta) = 0 \Rightarrow f = \mu_k N = \mu_k W_A \cos(\theta)$$

Work & Energy: (applied to the system)

$$\underbrace{K_1}_{\text{zero}} + U_{1 \rightarrow 2} = K_2 \quad \text{with} \quad U_{1 \rightarrow 2} = (U_{1 \rightarrow 2})_{\text{friction}} + (U_{1 \rightarrow 2})_{W_A} + (U_{1 \rightarrow 2})_{W_B}$$

where

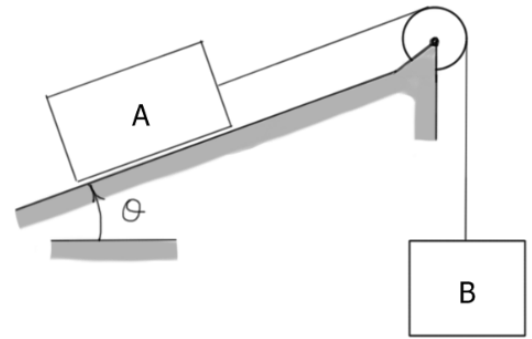
$$\left. \begin{aligned} (U_{1 \rightarrow 2})_{\text{friction}} &= -f d = -\mu_k W_A \cos(\theta) d \\ (U_{1 \rightarrow 2})_{W_A} &= -W_A d \sin(\theta) = V_1 - V_2 \\ (U_{1 \rightarrow 2})_{W_B} &= W_B d = V_1 - V_2 \end{aligned} \right\} U_{1 \rightarrow 2} = 95.2144 \text{ (ft-lb)}$$

Note that because $U_{1 \rightarrow 2} > 0$, it confirms that B **moves down**.

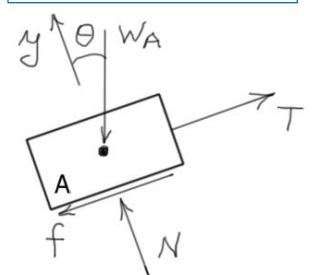
$$K_2 = \frac{1}{2} \left(\frac{W_A}{g} \right) v_A^2 + \frac{1}{2} \left(\frac{W_B}{g} \right) v_B^2 = \frac{1}{2} \left(\frac{W_A + W_B}{g} \right) v^2 \quad (\text{blocks have the same velocity})$$

Substituting into the **work & energy equation**

$$\frac{1}{2} \left(\frac{W_A + W_B}{g} \right) v^2 = U_{1 \rightarrow 2} \Rightarrow v = \sqrt{\frac{2gU_{1 \rightarrow 2}}{W_A + W_B}} \approx 5.40 \text{ (ft/s)}$$



Free body diagram



Observations:

1. We treated the two blocks and the connecting cable as a **single system**, and consequently, we had a **single work & energy equation**. The **net** work of the cable tension on the system is **zero**, so it **does not contribute** to this equation. (The work done by the cable tension on block A is positive (causing it to move up the plane), and the work it does on block B is of the same magnitude but negative (slowing its downward motion).)
2. If we treated the **two blocks separately**, we could write a **work & energy equation for each block**. The cable tension does **nonzero** work on each block, so the cable tension is an **unknown** in these two equations.