

ME 2580 Example #21: (Work & Energy)

Given: $W_{\text{box}} = W = 5 \text{ (lb)}$, $R = 2 \text{ (ft)}$, $v_A = 4 \text{ (ft/s)}$, surface is smooth (no friction)

Find: $\hat{\theta}$ the angle where the box begins to leave the surface

Solution: using work & energy and Newton's law

Newton's Law:

$$\checkmark \sum F_n = W \cos(\theta) - N = \left(\frac{W}{g}\right)\left(\frac{v^2}{R}\right)$$

At the point where the box leaves the surface, $N = 0$ and $\theta = \hat{\theta}$, so

$$W \cos(\hat{\theta}) = \left(\frac{W}{g}\right)\left(\frac{v^2}{R}\right) \Rightarrow v^2 = g R \cos(\hat{\theta})$$

Work & Energy: (applied to the box)

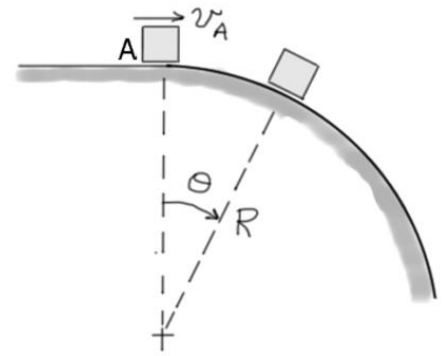
$$K_1 + U_{1 \rightarrow 2} = K_2 \quad \text{with} \quad U_{1 \rightarrow 2} = (U_{1 \rightarrow 2})_W = W(R - R \cos(\hat{\theta})) = V_1 - V_2$$

Substituting into the work and energy equation:

$$\frac{1}{2}\left(\frac{W}{g}\right)v_A^2 + W(R - R \cos(\hat{\theta})) = \frac{1}{2}\left(\frac{W}{g}\right)v_2^2$$

$$\Rightarrow \left(\frac{v_A^2}{2g}\right) + R - R \cos(\hat{\theta}) = \left(\frac{1}{2g}\right)g R \cos(\hat{\theta}) = \frac{1}{2} R \cos(\hat{\theta}) \Rightarrow \frac{3}{2} R \cos(\hat{\theta}) = \left(\frac{v_A^2}{2g}\right) + R$$

$$\Rightarrow \cos(\hat{\theta}) = \frac{2}{3R}\left(\frac{v_A^2}{2g} + R\right) = 0.749482 \Rightarrow \hat{\theta} \approx 41.5 \text{ (deg)}$$



Free body diagram

