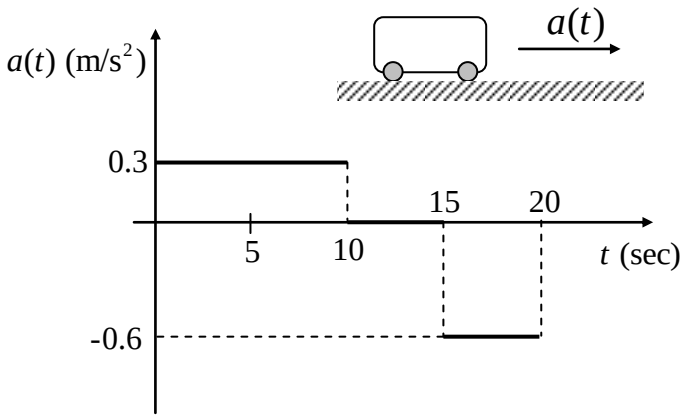
ME 2580 Dynamics
Acceleration Profiles



Given: A car has an acceleration profile as shown. It's initial position and velocity are zero. $\left(s(0)=v(0)=0\right)$

$$\boxed{v(t)=\int a(t)dt} \quad \text{and} \quad \boxed{s(t)=\int v(t)dt}$$

Find: (a) The velocity function $v(t)$; and (b) the displacement function $s(t)$ for the car for $0\le t\le 20$ (sec).

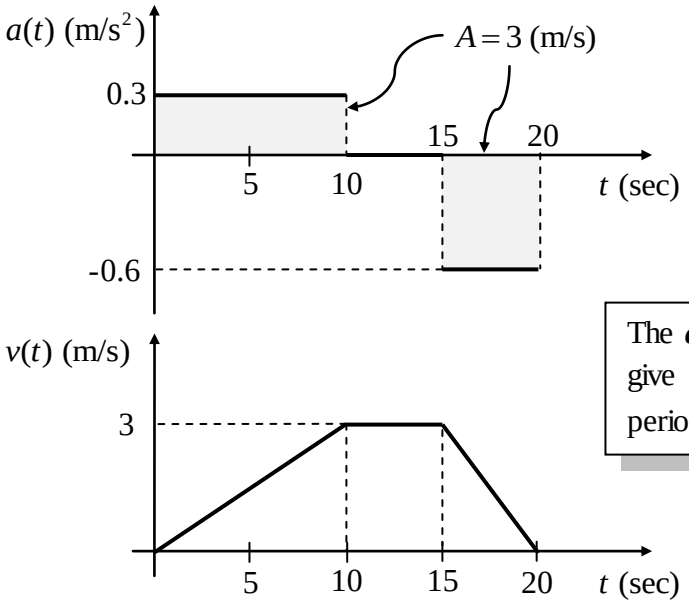
Solution:

(a) We can construct the velocity diagram easily from the acceleration diagram. When the acceleration is constant, the velocity varies linearly, and when the acceleration is zero, the velocity is constant.

$$\boxed{v(t)=\int 0.3dt=0.3t+D=0.3t} \text{ for } 0\le t\le 10 \quad (\text{recall that } v(0)=0)$$

$$\boxed{v(t)=\int 0dt=D=v(10)=3 \text{ (m/s)}} \text{ for } 10\le t\le 15$$

$$\boxed{v(t)=\int -0.6dt=-0.6t+D=-0.6t+12} \text{ for } 15\le t\le 20 \quad \left(v(15)=3 \text{ (m/s)}\right)$$



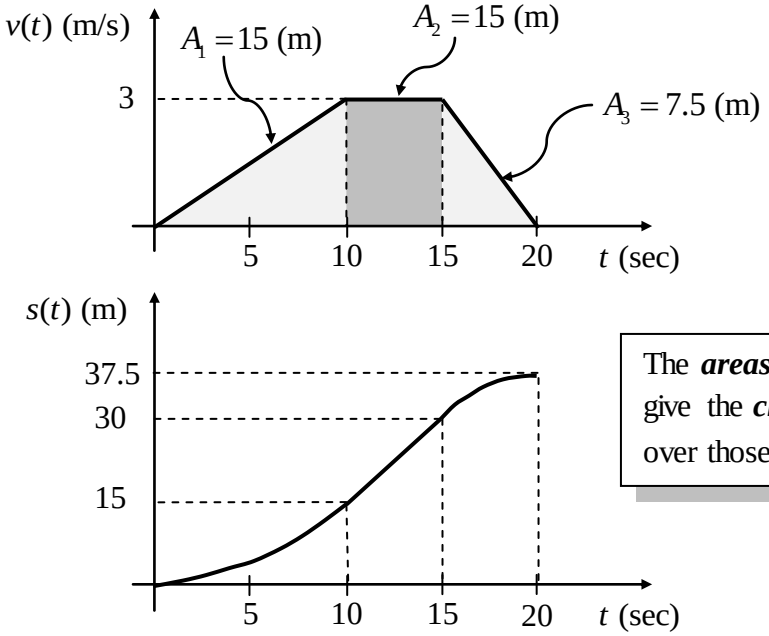
The ***areas*** under the acceleration profile give the ***changes*** in velocity over those periods of time

(b) The displacement function can now be derived from the velocity profile. When the velocity varies ***linearly***, the displacement will vary ***quadratically***, and when the velocity is ***constant***, the displacement will vary ***linearly***. The displacement changes are given by areas under the velocity function.

$$\boxed{s(t) = \int 0.3t\,dt = 0.15t^2 + D = 0.15t^2} \quad \text{for } 0 \le t \le 10 \quad \text{(recall that } s(0) = 0\text{)}$$

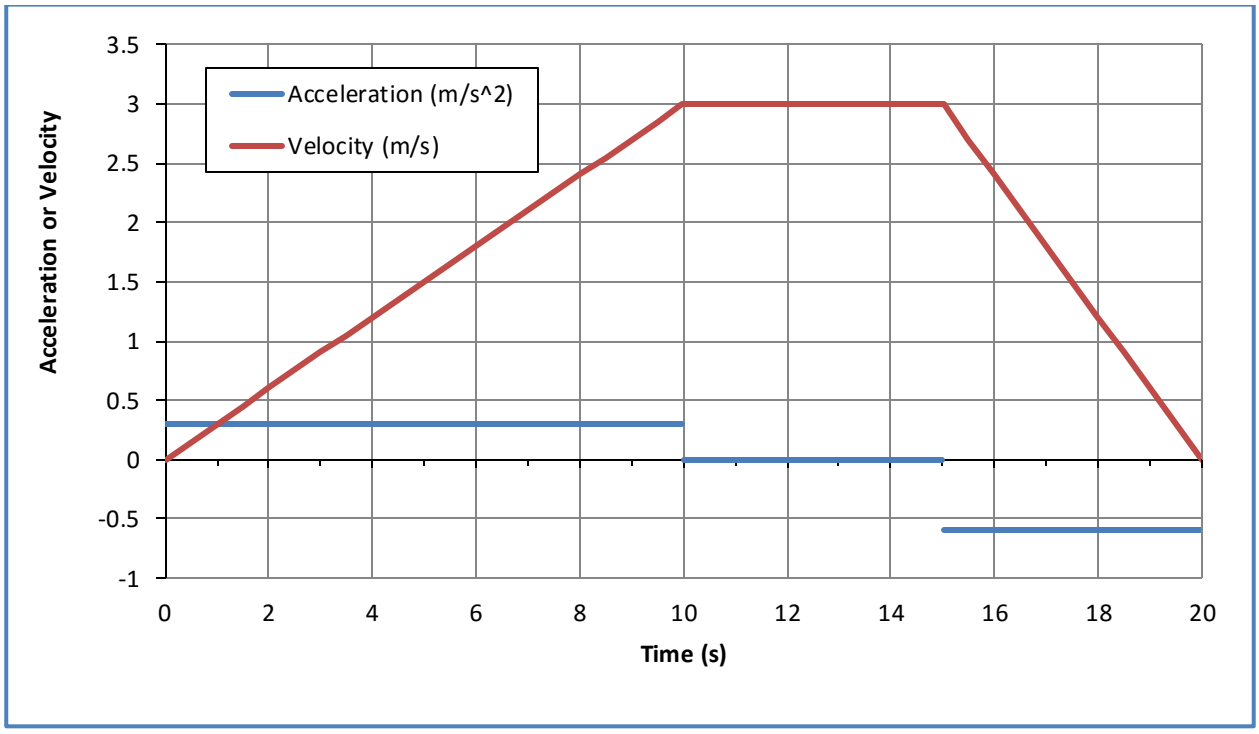
$$\boxed{s(t) = \int 3dt = 3t + D = 3t - 15} \quad \text{for } 10 \le t \le 15 \quad \big(s(10) = 15 \text{ (m)}\big)$$

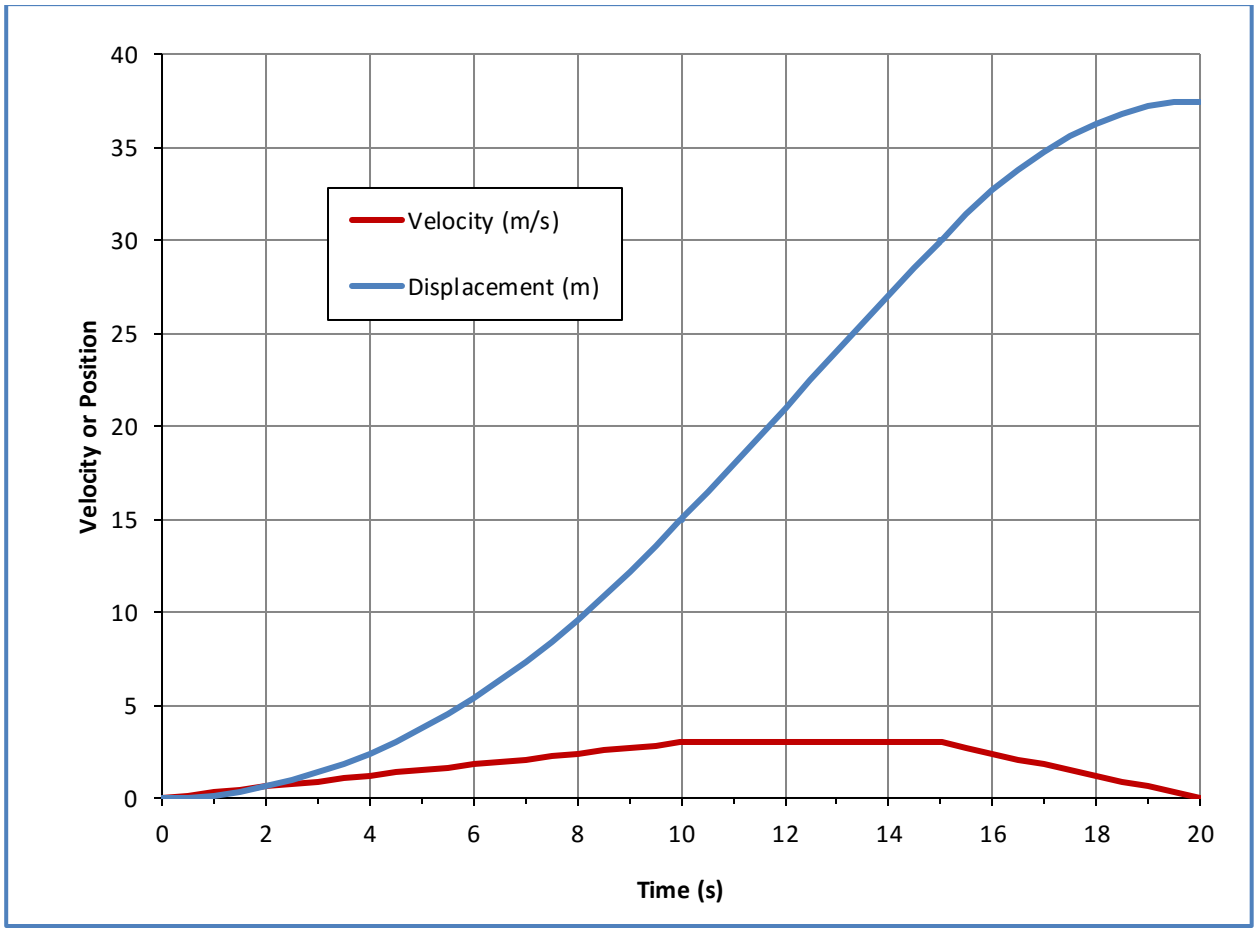
$$\boxed{\begin{aligned} s(t) &= \int -0.6t + 12\,dt = -0.3t^2 + 12t + D \\ &= -0.3t^2 + 12t - 82.5 \end{aligned}} \quad \text{for } 15 \le t \le 20 \quad \big(s(15) = 30 \text{ (m)}\big)$$



The ***areas*** under the velocity profile give the ***changes*** in displacement over those periods of time

In this case, note that the acceleration profile is a ***discontinuous*** function, the velocity profile is a ***piece-wise continuous*** function, and the displacement profile is a ***continuous*** function. If we do not need the actual functions, we can still construct the velocity and displacement profiles by simply measuring areas under the acceleration and velocity profiles.


Acceleration (m/s^2)
Velocity (m/s)
Acceleration or Velocity
Time (s)
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
0
2
4
6
8
10
12
14
16
18
20


Velocity (m/s)
Displacement (m)
Velocity or Position
Time (s)
40
35
30
25
20
15
10
5
0
0
2
4
6
8
10
12
14
16
18
20