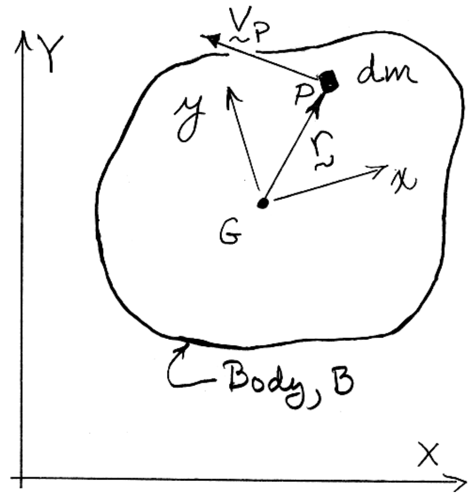
**ME 2580 Dynamics**
**Angular Momentum of a Rigid Body (2D)**




The figure depicts a rigid body moving in two dimensions. The ***X-Y*** axes are fixed, the ***x-y*** axes are fixed in (and move with) the body, and $G$ is the mass center of the body. The ***angular momentum*** of the body about its mass center $G$ is defined as

$$\underset{\sim}{H}_G = \int_B \left(\underset{\sim}{r} \times \underset{\sim}{v}_P\right) dm$$

Using the ***relative velocity equation***, the angular momentum can be written as

$$\underset{\sim}{H}_G = \int_B \underset{\sim}{r} \times \left(\underset{\sim}{v}_G + \underset{\sim}{v}_{P/G}\right) dm = \int_B \left(\underset{\sim}{r} \times \underset{\sim}{v}_G\right) dm + \int_B \left(\underset{\sim}{r} \times \underset{\sim}{v}_{P/G}\right) dm = \left(\int_B \underset{\sim}{r}\, dm\right) \times \underset{\sim}{v}_G + \int_B \underset{\sim}{r} \times \left(\underset{\sim}{\omega} \times \underset{\sim}{r}\right) dm$$

or

$$\boxed{\underset{\sim}{H}_G = \int_B \underset{\sim}{r} \times \left(\underset{\sim}{\omega} \times \underset{\sim}{r}\right) dm}$$

where

$$\underset{\sim}{\omega} \times \underset{\sim}{r} = \omega\, \underset{\sim}{k} \times \left(x'\, \underset{\sim}{i} + y'\, \underset{\sim}{j}\right) = \omega\left(-y'\, \underset{\sim}{i} + x'\, \underset{\sim}{j}\right)$$

$$\underset{\sim}{r} \times \left(\underset{\sim}{\omega} \times \underset{\sim}{r}\right) = \omega \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ x' & y' & 0 \\ -y' & x' & 0 \end{vmatrix} = \omega\left(x'^2 + y'^2\right)\underset{\sim}{k}$$

Substituting into the boxed equation above gives

$$\underset{\sim}{H}_G = \int_B \omega\left(x'^2 + y'^2\right)\underset{\sim}{k}\, dm = \left(\int_B \left(x'^2 + y'^2\right) dm\right)\omega \underset{\sim}{k} = I_G \underset{\sim}{\omega}$$

So, the ***angular momentum*** of a rigid body about its ***mass center*** is

$$\boxed{\underset{\sim}{H}_G = I_G\, \underset{\sim}{\omega}}$$

Recall that the ***linear momentum*** of a rigid body is $\boxed{\underset{\sim}{L} = m\, \underset{\sim}{v}_G}$.