ME 2580 Dynamics - Equation Sheet #1

Motion Tangent to Path:

$$s = s(t)$$
$$v = \frac{ds}{dt}$$
$$a = \frac{dv}{dt}$$

$$a = a(t)$$
$$\frac{dv}{dt} = a(t)$$
$$\int dv = \int a(t)\,dt$$
$$\int ds = \int v(t)\,dt$$

$$a = a(s)$$
$$v\frac{dv}{ds} = a(s)$$
$$\int v dv = \int a(s)\,ds$$

$$a = a(v)$$
$$\frac{dv}{dt} = a(v)$$
$$\int \frac{dv}{a(v)} = \int dt$$
$$\int ds = \int v(t)\,dt$$

$$a = a(v)$$
$$v\frac{dv}{ds} = a(v)$$
$$\int \frac{v dv}{a(v)} = \int ds$$

Constant Acceleration:

$$a(t) = a_0 = \text{constant}$$
$$v(t) = v_0 + a_0 t$$
$$s(t) = s_0 + v_0 t + \tfrac{1}{2} a_0 t^2$$
$$v^2 = v_0^2 + 2a_0(s - s_0)$$

Normal & Tangential Components:
($\underset{\sim}{e}_n$ and $\underset{\sim}{e}_t$)

$$\underset{\sim}{v} = v\underset{\sim}{e}_t$$
$$\underset{\sim}{a} = \dot{v}\underset{\sim}{e}_t + \left(\frac{v^2}{\rho}\right)\underset{\sim}{e}_n$$
$$v = \rho\dot{\theta}$$
$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{\left|d^2y/dx^2\right|}$$

Helpful Integral

$$\int\left(\frac{f'(x)}{f(x)}\right)dx = \ln(f(x))$$

Cylindrical Components:
($\underset{\sim}{e}_r$, $\underset{\sim}{e}_\theta$, and $\underset{\sim}{k}$)

$$\underset{\sim}{r} = r\underset{\sim}{e}_r + z\underset{\sim}{k}$$
$$\underset{\sim}{v} = \dot{r}\underset{\sim}{e}_r + \left(r\dot{\theta}\right)\underset{\sim}{e}_\theta + \dot{z}\underset{\sim}{k}$$
$$\underset{\sim}{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\underset{\sim}{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\underset{\sim}{e}_\theta + \ddot{z}\underset{\sim}{k}$$

Circular Motion:

$$v = R\dot{\theta} = R\omega$$
$$a_t = a_\theta = R\ddot{\theta} = R\dot{\omega} = R\alpha$$
$$a_n = -a_r = \frac{v^2}{\rho} = R\dot{\theta}^2 = R\omega^2$$

Relative Motion

$$\underset{\sim}{r}_{A/B} = \underset{\sim}{r}_A - \underset{\sim}{r}_B$$
$$\underset{\sim}{v}_{A/B} = \underset{\sim}{v}_A - \underset{\sim}{v}_B$$
$$\underset{\sim}{a}_{A/B} = \underset{\sim}{a}_A - \underset{\sim}{a}_B$$