

ME 2580 Dynamics – Equation Sheet #3

Linear Impulse and Momentum (for Systems of Particles)

$$\sum m_i (\underline{v}_i)_1 + \sum \int_{t_1}^{t_2} \underline{F}_i dt = \sum m_i (\underline{v}_i)_2$$

or

$$m(\underline{v}_G)_1 + \sum \int_{t_1}^{t_2} \underline{F}_i dt = m(\underline{v}_G)_2$$

$$\underline{F}_{avg} = \frac{1}{\Delta t} \sum \int_{t_1}^{t_2} \underline{F}_i dt = \frac{1}{\Delta t} I_{1 \rightarrow 2}$$

Conservation of Linear Momentum

$$\sum (m_i \underline{v}_i)_1 = \sum (m_i \underline{v}_i)_2$$

$$(\sum m_i) \underline{v}_G = \sum m_i \underline{v}_i = \text{constant}$$

Rigid Body Fixed Axis Rotation (2D)

$$\underline{\omega} = \omega \underline{k} = \dot{\theta} \underline{k} \quad \text{and} \quad \underline{\alpha} = \alpha \underline{k} = \dot{\omega} \underline{k} = \ddot{\theta} \underline{k}$$

$$\underline{v}_P = \underline{\omega} \times \underline{r}_P = r \dot{\theta} \underline{e}_\theta$$

$$\underline{a}_P = \frac{d}{dt} (\underline{\omega} \times \underline{r}_P) = (\underline{\alpha} \times \underline{r}_P) + \underline{\omega} \times (\underline{\omega} \times \underline{r}_P) = -r \dot{\theta}^2 \underline{e}_r + r \ddot{\theta} \underline{e}_\theta$$

Relative Motion of Two Points of a Rigid Body (2D)

$$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} = \underline{v}_B + (\underline{\omega} \times \underline{r}_{A/B})$$

$$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B} = \underline{a}_B + (\underline{\alpha} \times \underline{r}_{A/B}) - \omega^2 \underline{r}_{A/B}$$

Point Moving on a Rigid Body: Rotating Axes

$$\underline{v}_A = \underline{v}_B + \underline{v}_{Arel} + \underline{\omega} \times \underline{r}_{A/B} = \underline{v}_B + (\underline{v}_{A/B})_{xyz} + \underline{\Omega} \times \underline{r}_{A/B}$$

$$\underline{a}_A = \underline{a}_B + \underline{a}_{Arel} + 2(\underline{\omega} \times \underline{v}_{Arel}) + (\underline{\alpha} \times \underline{r}_{A/B}) - \omega^2 \underline{r}_{A/B}$$

$$= \underline{a}_B + (\underline{a}_{A/B})_{xyz} + 2(\underline{\Omega} \times (\underline{v}_{A/B})_{xyz}) + (\underline{\dot{\Omega}} \times \underline{r}_{A/B}) - \Omega^2 \underline{r}_{A/B}$$

Direct Central Impact of Two Particles

$$m_A (v_{Ax})_1 + m_B (v_{Bx})_1 = m_A (v_{Ax})_2 + m_B (v_{Bx})_2$$

$$e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}$$

Oblique Central Impact of Two Particles

$$(v_{Ay})_2 = (v_{Ay})_1 \quad \text{and} \quad (v_{By})_2 = (v_{By})_1$$

$$m_A (v_{Ax})_1 + m_B (v_{Bx})_1 = m_A (v_{Ax})_2 + m_B (v_{Bx})_2$$

$$e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}$$

Collisions with a Fixed Surface

$$(v_{Ay})_2 = (v_{Ay})_1 \quad \text{and} \quad (v_{Ax})_2 = -e(v_{Ax})_1$$

Constant Angular Acceleration

$$\alpha = \alpha_c = \text{constant}$$

$$\omega = \omega_0 + \alpha_c t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha_c t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha_c (\theta - \theta_0)$$