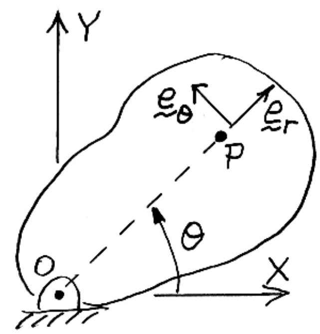
ME 2580 Dynamics
Pure Rotational Motion (Fixed Axis Rotation) in Two-Dimensions



A body *B* has ***pure rotational motion*** when one point of the body is fixed and all the other points of the body rotate around it. The body is rotating about an axis that passes through the fixed point and is normal to the plane of motion. In this case, all the points of the body have ***circular motion***.

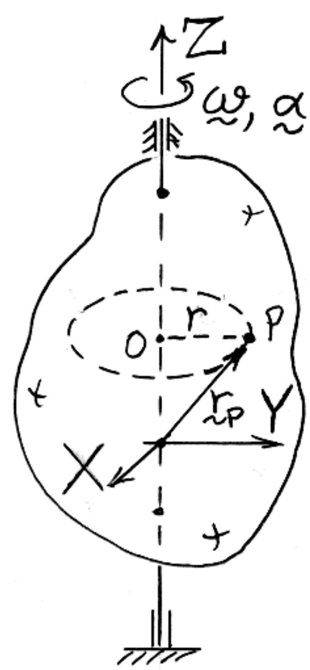
The kinematics of all points of *B* may be computed using the angular velocity and angular acceleration of *B*. They are defined as follows

$$\boxed{\underset{\sim}{\omega} = \frac{d\theta}{dt}\underset{\sim}{k} = \dot{\theta}\underset{\sim}{k}} \qquad \text{and} \qquad \boxed{\underset{\sim}{\alpha} = \dot{\omega}\underset{\sim}{k} = \ddot{\theta}\underset{\sim}{k}}.$$

Here, θ is measured ***counterclockwise*** relative to a ***fixed horizontal line***. The velocity and acceleration of point *P* may be computed as

$$\underset{\sim}{v}_P = \frac{d}{dt}\left(\underset{\sim}{r}_P\right) = \frac{d}{dt}\left(r\,\underset{\sim}{e}_r\right) = r\,\dot{\underset{\sim}{e}}_r = r\left(\dot{\theta}\underset{\sim}{k}\times\underset{\sim}{e}_r\right) = \dot{\theta}\underset{\sim}{k}\times\left(r\underset{\sim}{e}_r\right) \qquad \text{OR} \qquad \boxed{\underset{\sim}{v}_P = \underset{\sim}{\omega}\times\underset{\sim}{r}_P = r\,\dot{\theta}\,\underset{\sim}{e}_\theta}$$

$$\boxed{\underset{\sim}{a}_P = \frac{d}{dt}\left(\underset{\sim}{\omega}\times\underset{\sim}{r}_P\right) = \left(\underset{\sim}{\alpha}\times\underset{\sim}{r}_P\right) + \underset{\sim}{\omega}\times\left(\underset{\sim}{\omega}\times\underset{\sim}{r}_P\right) = -r\,\dot{\theta}^2\underset{\sim}{e}_r + r\,\ddot{\theta}\,\underset{\sim}{e}_\theta}$$



Usually, the velocity and acceleration are computed directly using the ***radial*** and ***transverse*** coordinates (r,θ) as shown in the boxed equations above. However, if the plane of motion is not easily identified, they may be calculated using the vector expressions. For example, the velocity and acceleration of the point *P* of the rotating three-dimensional shape may be calculated as

$$\boxed{\underset{\sim}{v}_P = \underset{\sim}{\omega}\times\underset{\sim}{r}_P} \qquad \text{and} \qquad \boxed{\underset{\sim}{a}_P = \left(\underset{\sim}{\alpha}\times\underset{\sim}{r}_P\right) + \underset{\sim}{\omega}\times\left(\underset{\sim}{\omega}\times\underset{\sim}{r}_P\right)}$$

where $\underset{\sim}{r}_P$ is the position vector of *P* relative to ***any*** point on the axis of rotation.