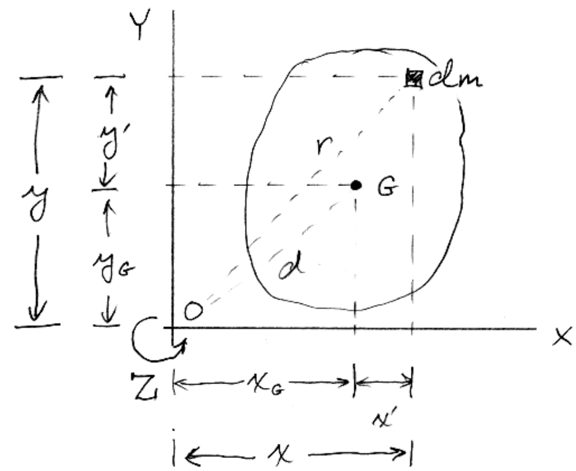
ME 2580 Dynamics
Mass Moments of Inertia (2D)

Definition of Mass Moment of Inertia



The figure depicts a rigid body in two dimensions. The moment of inertia of the body about the *Z*-axis is defined as

$$I_Z = \int_B r^2 dm = \int_B \left(x^2 + y^2\right) dm \,.$$

The moment of inertia I_Z measures the distribution of mass about the *Z*-axis. The ***larger*** the inertia, the ***further*** the mass is spread away from the axis. The ***smaller*** the inertia, the ***closer*** the mass is located to the axis. The units of I_Z are ***slug-ft²*** or ***kg-m²***.

Parallel Axes Theorem

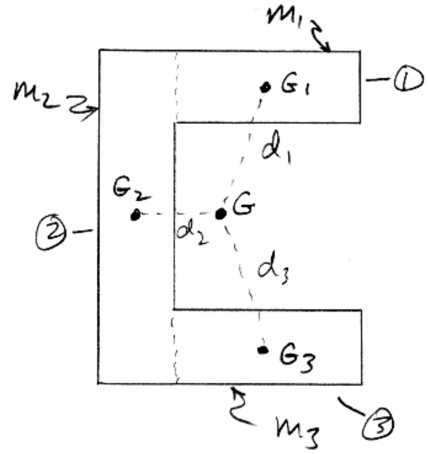
The moment of inertia of the body about the *Z*-axis can be related to the moment of inertia about an axis parallel to *Z* and passing through *G* as follows:

$$\begin{aligned}
I_Z &= \int_B \left(x^2 + y^2\right) dm = \int_B \left(\left(x_G + x'\right)^2 + \left(y_G + y'\right)^2\right) dm \\
&= \int_B \left(x_G^2 + y_G^2\right) dm + \int_B 2 x_G x' dm + \int_B 2 y_G y' dm + \int_B \left(x'^2 + y'^2\right) dm \\
&= \left(x_G^2 + y_G^2\right) \int_B dm + 2 x_G \left(\int_B x' dm \right) + 2 y_G \left(\int_B y' dm \right) + \left(I_Z\right)_G \\
&= M\, d^2 + \left(I_Z\right)_G
\end{aligned}$$

or

$$\boxed{I_Z = \left(I_Z\right)_G + M\, d^2} \qquad \text{or} \qquad \boxed{I_O = I_G + M\, d^2} \qquad \text{(Parallel Axes Theorem)}$$

Mass Moments of Inertia for Composite Shapes



The mass moment of inertia of a composite shape is the sum of the inertias of the individual component shapes. If G is the mass center of the composite shape, then

$$I_G = \sum_i \left(I_G\right)_i$$

where $\left(I_G\right)_i$ represents the moment of inertia of the i^{th} component shape about the composite mass center G and may be calculated as follows

$$\left(I_G\right)_i = \left(I_{G_i}\right) + m_i d_i^2 .$$

Here, I_{G_i} represents the moment of inertia of the i^{th} component shape about its own mass center G_i and can usually be found in the tables for common geometric shapes.