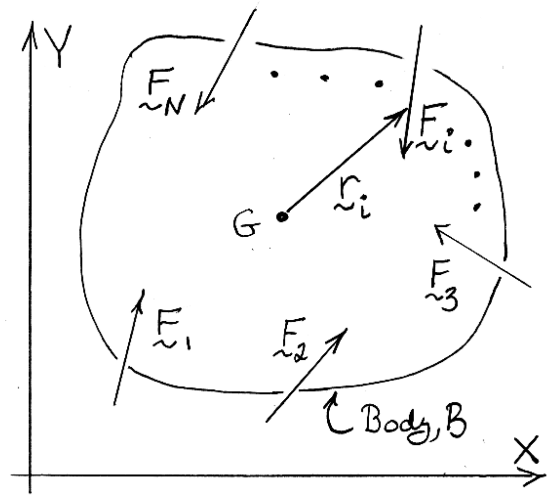
ME 2580 Dynamics
Newton's Law for Rigid Body Motion (2D)

General Plane Motion

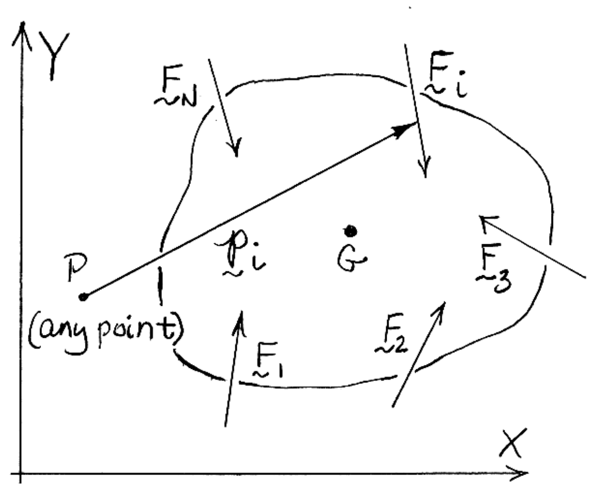
The figure depicts a rigid body moving in two dimensions. The motion is caused by a series of N forces $\underset{\sim}{F}_i \ (i = 1,\ldots,N)$. Generally, each force has the effect of both translating and rotating the body. Newton's laws of translational and rotational motion are



$$\sum_i \underset{\sim}{F}_i = m\underset{\sim}{a}_G$$
$$\sum_i (\underset{\sim}{M}_G)_i = \sum_i (\underset{\sim}{r}_i \times \underset{\sim}{F}_i) = I_G \underset{\sim}{\alpha}$$

Here, $(\underset{\sim}{M}_G)_i$ represents the ***moment*** of force $\underset{\sim}{F}_i$ about the ***mass center*** G, and I_G represents the ***moment of inertia*** of the body about a Z axis passing through G.

Note that the equation for rotational motion moment as written in the boxed equation above requires that that moments be taken about the mass center G. If moments are to be taken about any point other than G, Newton's laws of translational and rotational motion may be written as



$$\sum_i \underset{\sim}{F}_i = m\underset{\sim}{a}_G$$
$$\sum_i (\underset{\sim}{M}_P)_i = \sum_i (\underset{\sim}{p}_i \times \underset{\sim}{F}_i) = I_G \underset{\sim}{\alpha} + \left(\underset{\sim}{r}_{G/P} \times m\underset{\sim}{a}_G\right)$$

The term $\underset{\sim}{r}_{G/P} \times m\underset{\sim}{a}_G$ that is added to the right hand side of the moment equation represents the moment of $m\underset{\sim}{a}_G$ about the point P. The vector $m\underset{\sim}{a}_G$ is assumed to act through G.

Special Cases:

Pure Translational Motion:

When a body is undergoing pure translational motion, then the equations of motion may be written as

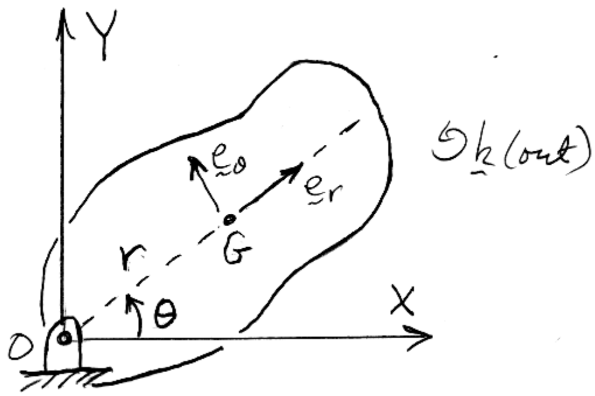
$$\sum_i \underset{\sim}{F}_i = m\underset{\sim}{a}_G \qquad \sum_i (\underset{\sim}{M}_G)_i = \sum_i (\underset{\sim}{r}_i \times \underset{\sim}{F}_i) = 0$$

or

$$\sum_i \underset{\sim}{F}_i = m\underset{\sim}{a}_G \qquad \sum_i (\underset{\sim}{M}_P)_i = \sum_i (\underset{\sim}{p}_i \times \underset{\sim}{F}_i) = \left(\underset{\sim}{r}_{G/P} \times m\underset{\sim}{a}_G\right)$$

Fixed Axis Rotation:

When a body is undergoing fixed axis rotation as shown by the figure at the right, then the equations of motion can be written as



$$\sum_i \underset{\sim}{F}_i = m\underset{\sim}{a}_G = m\left(r\alpha\underset{\sim}{e}_\theta - r\omega^2\underset{\sim}{e}_r\right) \qquad \sum_i (\underset{\sim}{M}_O)_i = \sum_i (\underset{\sim}{r}_i \times \underset{\sim}{F}_i) = I_O\,\underset{\sim}{\alpha}$$

Here I_O represents the ***moment of inertia*** of the body about a Z axis passing through the ***fixed point*** O.