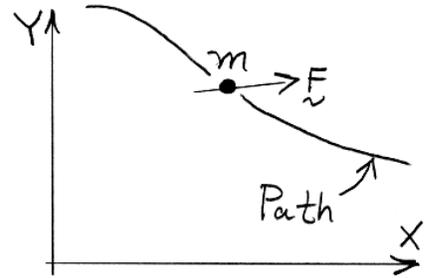


ME 2580 Dynamics

Principle of Impulse and Momentum for Particles

Recall that Newton's second law for a particle may be written as $\underline{F} = m\underline{a}$, where \underline{F} is the **resultant** force acting on the particle and \underline{a} is its **acceleration**. Recall also that the force and the acceleration are generally not tangent to the path, except in the case of rectilinear motion.



Previously, we developed a **scalar, integrated form** of Newton's second law, namely the **principle of work and energy**. Here we develop a **vector, integrated form** of Newton's second law called the **principle of impulse and momentum**. Integrating Newton's law with respect to **time** gives:

$$\int_{t_1}^{t_2} \underline{F} dt = \underline{L}_{1 \rightarrow 2} = m \int_{t_1}^{t_2} (d\underline{v}/dt) dt = m\underline{v}_2 - m\underline{v}_1 = \Delta \underline{L} \quad \text{or} \quad \boxed{m\underline{v}_1 + \int_{t_1}^{t_2} \underline{F} dt = m\underline{v}_2}$$

This is the **principle of linear impulse and momentum**. It states that the **resultant linear impulse** on the system over the **time interval** from $t_1 \rightarrow t_2$ gives rise to a **change in linear momentum**. Note that this equation, like Newton's law, is a **vector** equation. Also, note that the term "**linear**" means "**translational**" as opposed to "**angular**" or "**rotational**".

This principle can be extended to a system of particles by noting that the **net** impulse of all equal and opposite internal forces in a system is **zero**. So, in this case, for a system of particles, we have

$$\boxed{\sum m_i (\underline{v}_i)_1 + \sum \int_{t_1}^{t_2} \underline{F}_i dt = \sum m_i (\underline{v}_i)_2} \quad \text{or} \quad \boxed{m(\underline{v}_G)_1 + \sum \int_{t_1}^{t_2} \underline{F}_i dt = m(\underline{v}_G)_2}$$

Here, $\sum \int_{t_1}^{t_2} \underline{F}_i dt$ represents the **sum** of the impulses of all **external forces** and the **net sum** of the impulses of all internal forces acting on the system, and G represents the **mass center** of the system. The **time-averaged, net force** acting on the system is defined to be

$$\boxed{\underline{F}_{avg} = \frac{1}{\Delta t} \sum \int_{t_1}^{t_2} \underline{F}_i dt = \frac{1}{\Delta t} \underline{L}_{1 \rightarrow 2}}$$

Definitions: Forces that are large, act over a short interval of time, and cause **significant** changes in momentum are called **impulsive** forces. Forces that cause **insignificant** changes in momentum over short intervals of time are called **non-impulsive** forces. **Weight forces** are generally considered to be **non-impulsive**.