

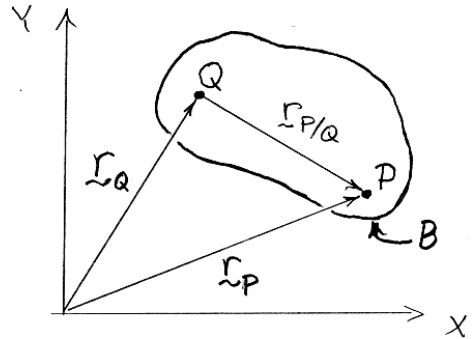
## ME 2580 Dynamics

### Relative Acceleration of Two Points Fixed on a Rigid Body

The figure depicts a rigid body moving in two dimensions. The two points  $P$  and  $Q$  are **fixed** on the body. At any instant of time, the position vector of  $P$  may be written as

$$\underline{r}_P = \underline{r}_Q + \underline{r}_{P/Q}$$

Here,  $\underline{r}_{P/Q}$  is called the position vector of  $P$  relative to  $Q$ .



In previous notes, it was shown that this equation could be differentiated to find a relationship between the velocities of  $P$  and  $Q$ . In particular, it was shown

$$\underline{v}_P = \underline{v}_Q + \underline{v}_{P/Q} = \underline{v}_Q + (\underline{\omega} \times \underline{r}_{P/Q}) \quad (\text{relative velocity equation})$$

Here,  $\underline{v}_{P/Q}$  is the velocity of  $P$  relative to  $Q$  which is **the velocity of  $P$  assuming that  $Q$  is fixed**. Similarly, the accelerations of points  $P$  and  $Q$  can be related by **differentiating** the relative velocity equation.

$$\underline{a}_P = \frac{d}{dt}(\underline{v}_Q + (\underline{\omega} \times \underline{r}_{P/Q})) = \frac{d\underline{v}_Q}{dt} + \frac{d}{dt}(\underline{\omega} \times \underline{r}_{P/Q}) = \underline{a}_Q + \left(\frac{d\underline{\omega}}{dt} \times \underline{r}_{P/Q}\right) + \left(\underline{\omega} \times \frac{d\underline{r}_{P/Q}}{dt}\right)$$

or

$$\underline{a}_P = \underline{a}_Q + \underline{a}_{P/Q} = \underline{a}_Q + (\underline{\alpha} \times \underline{r}_{P/Q}) + (\underline{\omega} \times \underline{v}_{P/Q}) = \underline{a}_Q + (\underline{\alpha} \times \underline{r}_{P/Q}) + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{P/Q})$$

In **two dimensions**, the triple vector product  $\underline{\omega} \times (\underline{\omega} \times \underline{r}_{P/Q}) = -\omega^2 \underline{r}_{P/Q}$ , so we can write

$$\underline{a}_P = \underline{a}_Q + \underline{a}_{P/Q} = \underline{a}_Q + (\underline{\alpha} \times \underline{r}_{P/Q}) - \omega^2 \underline{r}_{P/Q}$$

Here,  $\underline{a}_{P/Q}$  is the acceleration of  $P$  relative to  $Q$  which is the **acceleration of  $P$  assuming that  $Q$  is fixed**. It may be written in terms of  $\underline{e}_r$  and  $\underline{e}_\theta$  as follows.

$$\underline{a}_{P/Q} = (\underline{a}_{P/Q})_t + (\underline{a}_{P/Q})_n = L\alpha \underline{e}_\theta - L\omega^2 \underline{e}_r$$

Here,  $L$  is the **distance** between  $P$  and  $Q$ .

