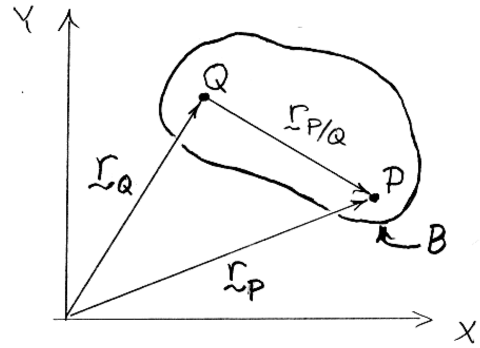
ME 2580 Dynamics
Relative Velocity of Two Points Fixed on a Rigid Body



The figure depicts a rigid body moving in two dimensions. The two points P and Q are ***fixed*** on the body. At any instant of time, the position vector of P may be written as

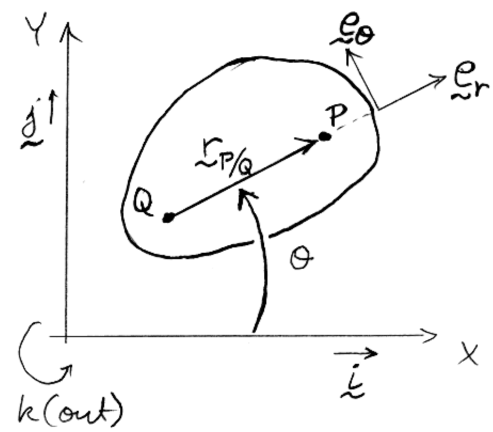
$$\boxed{\underset{\sim}{r}_P = \underset{\sim}{r}_Q + \underset{\sim}{r}_{P/Q}}$$

Here, $\underset{\sim}{r}_{P/Q}$ is called the position vector of P ***relative*** to Q.

The ***velocities*** of P and Q may be related by ***differentiating*** the above equation

$$\frac{d}{dt}\left(\underset{\sim}{r}_P\right) = \frac{d}{dt}\left(\underset{\sim}{r}_Q\right) + \frac{d}{dt}\left(\underset{\sim}{r}_{P/Q}\right) \quad \text{or} \quad \boxed{\underset{\sim}{v}_P = \underset{\sim}{v}_Q + \frac{d}{dt}\left(\underset{\sim}{r}_{P/Q}\right)}$$

Note that because the body is rigid, the position vector $\underset{\sim}{r}_{P/Q}$ has ***constant length***. However, the derivative of $\underset{\sim}{r}_{P/Q}$ is ***not zero***, because it will ***change direction*** as the body rotates.



To calculate the derivative of $\underset{\sim}{r}_{P/Q}$, consider the figure at the right. Given that $\underset{\sim}{e}_r$ is a unit vector pointed from Q towards P, the position vector $\underset{\sim}{r}_{P/Q}$ may be written as

$$\boxed{\underset{\sim}{r}_{P/Q} = L\underset{\sim}{e}_r}$$

where L represents the distance from Q to P. The derivative of $\underset{\sim}{r}_{P/Q}$ may be calculated as follows

$$\frac{d}{dt}\left(\underset{\sim}{r}_{P/Q}\right) = \frac{d}{dt}\left(L\underset{\sim}{e}_r\right) = L\dot{\underset{\sim}{e}}_r = L\left(\dot{\theta}\underset{\sim}{k}\right)\times\underset{\sim}{e}_r = \dot{\theta}\underset{\sim}{k}\times L\underset{\sim}{e}_r = \underset{\sim}{\omega}\times\underset{\sim}{r}_{P/Q}$$

or

$$\frac{d}{dt}\left(\underset{\sim}{r}_{P/Q}\right) = \underset{\sim}{\omega}\times\underset{\sim}{r}_{P/Q} = \dot{\theta}\underset{\sim}{k}\times L\underset{\sim}{e}_r = L\dot{\theta}\underset{\sim}{e}_\theta$$

This last result can now be combined with the boxed equation above to give

$$\boxed{\underset{\sim}{v}_P = \underset{\sim}{v}_Q + \left(\underset{\sim}{\omega}\times\underset{\sim}{r}_{P/Q}\right) = \underset{\sim}{v}_Q + \underset{\sim}{v}_{P/Q}}$$

This last equation defines $\underset{\sim}{v}_{P/Q}$ the ***velocity of P relative to Q***. Generally, the equation is used to relate the velocity of two points fixed on the same rigid body.