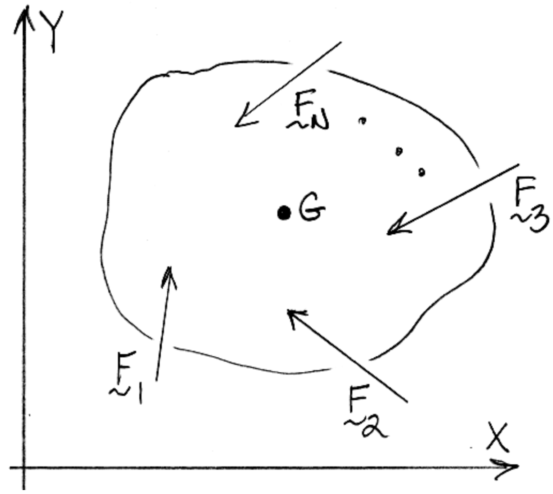
ME 2580 Dynamics
Work and Energy Principles for Rigid Body Motion (2D)

Principle of Work and Energy for a Single Body

The figure shows planar motion of a rigid body under the action of many forces. As a result of work done on the body by all the forces and torques, it will experience changes in ***kinetic energy***. The ***principle of work and energy*** states



$$\boxed{KE_1 + U_{1\to 2} = KE_2}$$

where $U_{1\to 2}$ represents the ***work done*** by all the forces and torques acting on the body as it moves from position 1 to position 2, and KE_i $(i = 1, 2)$ represent the kinetic energies of the body in positions 1 and 2, respectively.

The kinetic energy of a rigid body is defined to be

$$KE = \int_B \tfrac{1}{2}(\underset{\sim}{v}_P \cdot \underset{\sim}{v}_P)\,dm$$

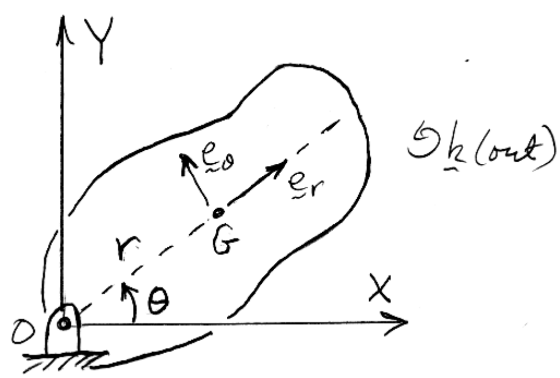
where P represents a generic point within the body. If the body is undergoing general plane motion, then it can be shown that

$$\boxed{KE = (KE)_{\text{translation}} + (KE)_{\text{rotation}} = \tfrac{1}{2} m v_G^2 + \tfrac{1}{2} I_G \omega^2} \qquad \text{(general plane motion)}$$

However, if the body is undergoing ***fixed-axis rotation*** about point O, then the kinetic energy may be written as

$$\boxed{KE = \tfrac{1}{2} I_O \omega^2} \qquad \text{(fixed-axis rotation)}$$

where I_O is the moment of inertia of the body about the axis of rotation.



Principle of Work and Energy for Systems of Bodies

The principle of work and energy can also be applied to a ***system of bodies***. As for a single body, the principle states

$$\boxed{KE_1 + U_{1\to 2} = KE_2}$$

where $U_{1\to 2}$ represents the work done by all the forces and torques acting on the ***system*** of bodies as they move from position 1 to position 2, and KE_i $(i = 1, 2)$ represent the kinetic energies of the ***system*** of bodies in positions 1 and 2, respectively. The kinetic energy of a system of bodies at any time is simply the ***sum*** of the kinetic energies of the individual bodies of the system at that time

$$\boxed{KE = \sum_{\text{bodies}} \left(\tfrac{1}{2} m v_G^2 + \tfrac{1}{2} I_G \omega^2\right)}$$

It is important when applying the principle of work and energy to a system of bodies to know ***which forces and torques do work and which do not***.

Work Done by External Forces on the System

Conservative Forces:

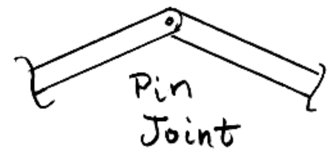
Forces applied to the system at locations that have ***nonzero displacement*** as the system moves from position 1 to position 2 do ***nonzero work***. Conversely, forces applied to the system at locations that have ***zero (net) displacement*** as the system moves from position 1 to position 2 do ***no work***.

Nonconservative Forces:

Forces applied to the system at locations that have ***nonzero displacement*** as the system moves from position 1 to position 2 do ***nonzero work***. Forces applied to the system at locations that move, but have ***zero (net) displacement*** as the system moves from position 1 to position 2 also do ***nonzero work***. ***Nonconservative forces do no work only when they are applied at points that do not move.***

Work Done by Internal Pin Forces

As stated above, the work done by forces at fixed support points do no work, because those points do not move. The work done by forces at pin joints that move is ***nonzero*** on each of the members, but the ***net work*** done on the system of the two members is ***zero***, because the pin forces on the members are equal and opposite and move through the same displacement.



Work Done by External Torques

Conservative Torques:

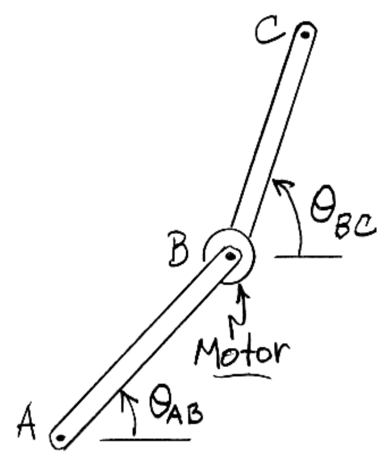
Torques applied to the system at locations that have ***nonzero rotation*** as the system moves from position 1 to position 2 do ***nonzero work***. Conversely, torques applied to the system at locations that have ***zero (net) rotation*** as the system moves from position 1 to position 2 do ***no work***.

Nonconservative Torques:

Torques applied to the system at locations that have ***nonzero rotation*** as the system moves from position 1 to position 2 do ***nonzero work***. Torques applied to the system at locations that rotate, but have ***zero (net) displacement*** as the system moves from position 1 to position 2 also do ***nonzero work***. ***Nonconservative torques do no work only when they are applied at points that do not rotate.***

Work Done by Torques Through a Pin Joint

The work done by a torque acting at a pin joint between two bodies is generally ***nonzero***. As an example, consider a motor located between two segments of a robot arm as shown in the diagram. The net work done by the motor on the arm is the sum of the work done on each of the segments (*AB* and *BC*)



$$U_{1\to 2} = \int_{(\theta_{AB})_1}^{(\theta_{AB})_2} (-M)\,d\theta + \int_{(\theta_{BC})_1}^{(\theta_{BC})_2} (M)\,d\theta \neq 0$$

Note that if the motor torque is *constant*, then the above equation reduces to

$$U_{1\to 2} = M(\Delta\theta_{BC} - \Delta\theta_{AB})$$

Work Done by Conservative Forces/Torques

If a force or torque is ***conservative*** (e.g. springs and gravity), then $U_{1\to 2}$ may be calculated using a ***potential energy*** function

$$\boxed{U_{1\to 2} = V_1 - V_2}$$

where

$$\boxed{V_{\substack{\text{translational}\\ \text{spring}}} = \tfrac{1}{2} k e^2} \qquad \boxed{V_{\substack{\text{rotational}\\ \text{spring}}} = \tfrac{1}{2} k \theta^2} \qquad \boxed{V_{\text{gravity}} = m g h_G}$$

Note that, as before, the potential energy associated with gravity is defined ***relative to a fixed (and arbitrary) datum***.

Principle of Conservation of Energy

If all the forces and torques acting on a system are ***conservative***, then the principle of work and energy may be written as

$$KE_1 + U_{1\to 2} = KE_1 + V_1 - V_2 = KE_2$$

or

$$\boxed{KE_1 + V_1 = KE_2 + V_2 = \text{constant}}$$