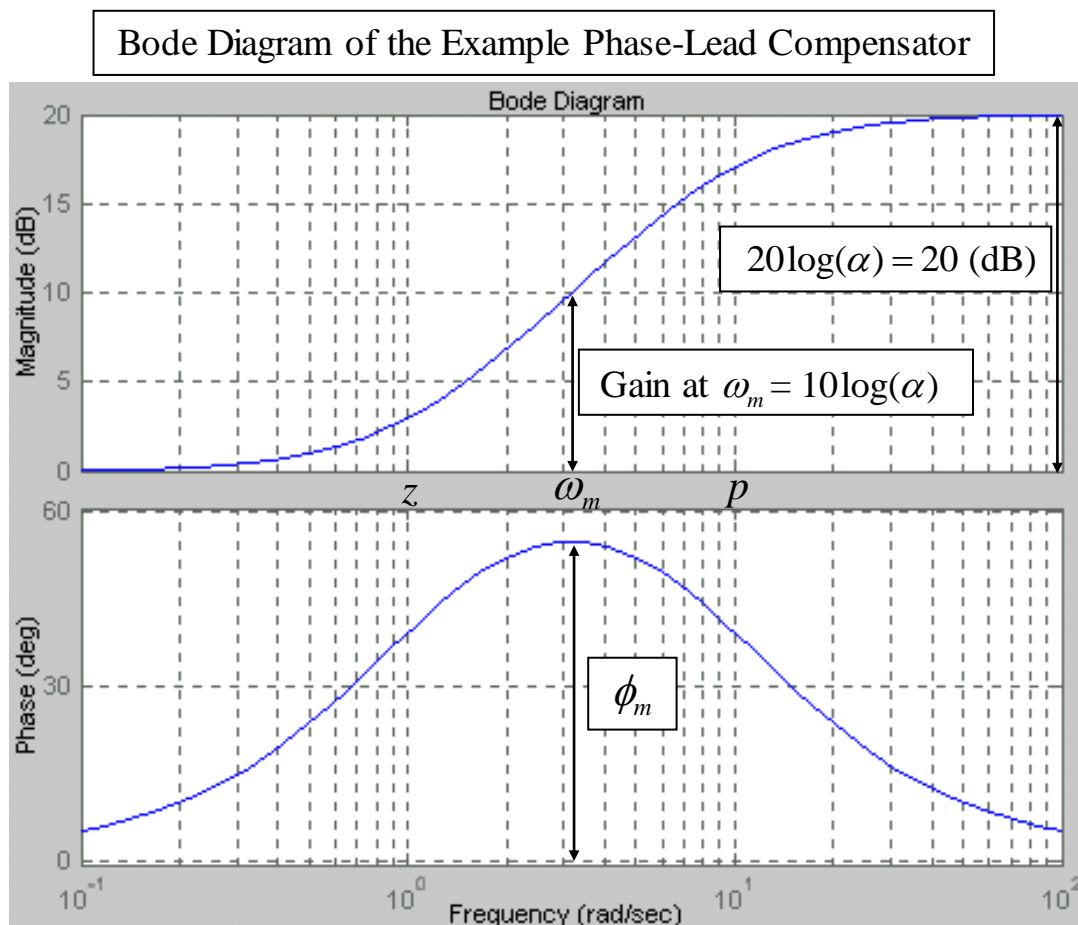


ME 4710 Motion and Control: Phase-Lead Compensator Design

Reference: Dorf & Bishop, Modern Control Systems, 10th Ed, Prentice-Hall, 2005.

Phase-Lead Compensator (PD type compensator)

- Transfer function of a *phase-lead* compensator: $G_c(s) = \alpha \left(\frac{s+z}{s+p} \right)$, where $|z| < |p|$
- The multiplier $\alpha = |p|/|z|$.
- As a result, the compensator *amplifies* at frequencies near and above the location of the zero, and *increases* the phase of the system near the *pole* and *zero* locations, as well.
- A Bode diagram of the phase-lead compensator $G_c(s) = 10 \left(\frac{s+1}{s+10} \right)$ is shown below.



- Here, $\alpha = 10$, the *logarithmic mean frequency* is $\omega_m = \sqrt{|pz|} = 3.16$ (rad/s), the overall magnitude increase is $20\log(\alpha) = 20$ (dB), and the phase shift at the mean frequency is

$$\phi_m = \sin^{-1} \left(\frac{\alpha - 1}{\alpha + 1} \right) = +54.9 \text{ (deg)}$$

Phase-Lead Compensator Design Using Bode Diagrams

- **Find loop gain K** required to satisfy the *steady-state error requirement* (if given).
- Evaluate the **phase margin (PM)** of the *uncompensated system* with the loop gain K to determine if proportional control is sufficient.
- **Find ϕ_a** the *necessary additional phase* required to give the desired phase margin.
- **Find α** for the *compensator* using the equation $\alpha = \frac{1 + \sin(\phi_a)}{1 - \sin(\phi_a)}$.
- Examine the **Bode plot** of the *uncompensated system* (with the loop gain K) to find the frequency where $M = -10\log(\alpha)$. Define this frequency to be ω_m the *logarithmic mean frequency* of the compensator. This will be the new **zero dB crossover** point.
- **Find the pole and zero** locations $p = \omega_m\sqrt{\alpha}$ and $z = p/\alpha$, and define the compensator to be $G_c(s) = K\alpha \left(\frac{s+z}{s+p} \right)$.
- **Check the phase margin** of the *compensated system* to see if the desired value has been attained. If not, then decide on the additional phase required, and repeat the steps above starting with the calculation of α .
- **Simulate** the time-domain performance.

Phase-Lead Compensator Design Using Root Locus Diagrams

- **Target Regions:** Set *damping ratio* (ζ) and *natural frequency* (ω_n) values for the complex poles (assuming they are all dominant and have no influence from transfer function zeros) to target a *desirable percent overshoot* and *settling time*.
- **Examine the uncompensated root locus diagram** to see if the pole locations determined above can be met with only proportional control.
- **Add a zero and pole** to $GH(s)$ modifying the root locus diagram to move branches into the target region.
 - Changes the **shape** of the root locus diagram in predictable ways.
 - Can change the locations of real poles of the closed loop system.
 - Separation between the pole and zero will **move all asymptotes to the left**.
 - The location of any **closed-loop zeros** may cause overshoot problems.
- **Simulate** the time-domain performance.