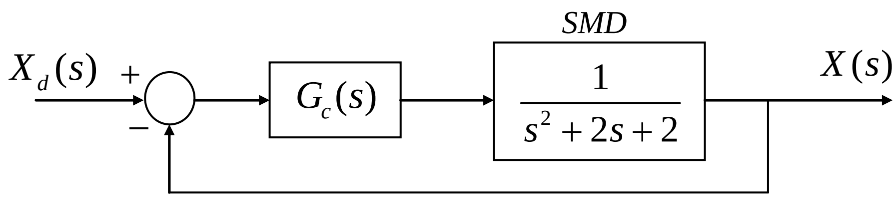
ME 4710 Motion and Control
Frequency Response Design of a Phase-Lead Compensator for a Spring-Mass-Damper (SMD) Positioning System

- To illustrate the ***frequency response design of a phase-lead compensator***, consider the following SMD positioning system controlled by the compensator $G_c(s)$. Here, $X_d(s)$ and $X(s)$ are the ***desired*** and ***actual positions*** of the mass.



- Proportional control ($G_c(s) = K$): ***Large gain*** is required to control steady-state error of a step response. Unfortunately, large gains produce ***undesirable***, ***oscillatory*** closed-loop response. Below, we design a ***phase-lead*** compensator to control the steady-state error and give desirable transient response.

Problem: Design a phase-lead compensator so the closed-loop system has a ***steady-state position error*** $e_{ss} = 1 - x_{ss} < 0.05$ to a unit step input and a ***phase margin*** $PM = 45$ (deg). Plot the step response of the resulting closed-loop system.

Frequency Response Design M-file: PhaseLeadPositionControlSMDwithBode.m

Step 1: Determine the ***required compensator gain*** to satisfy the ***error specification.***

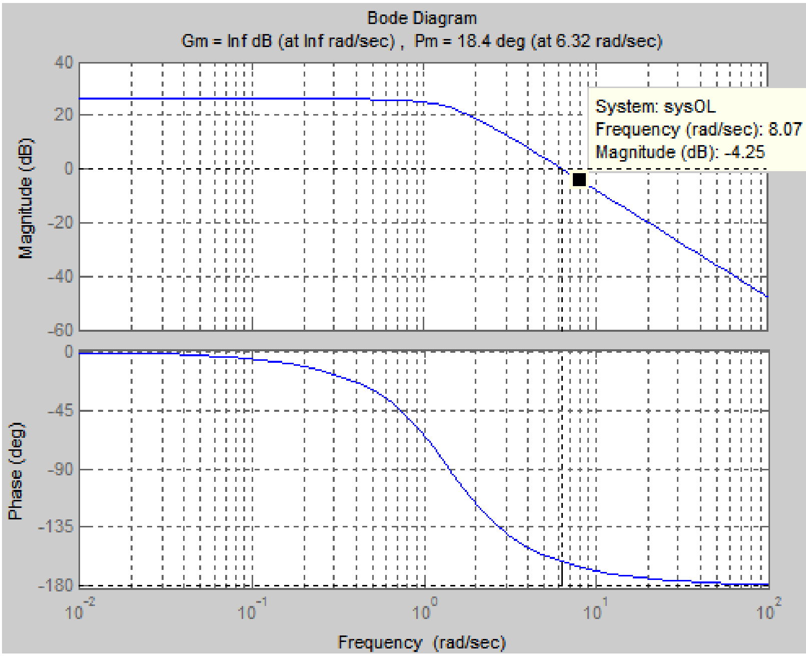
The steady state error can be defined in terms of the loop transfer function as

$$e_{ss} = \lim_{s\to 0}\left[\frac{1}{1+GH(s)}\right] = \frac{1}{1+K/2} = \frac{1}{1+K_p} < 0.05$$

So, $K_p > 19$ and $K > 38$. Let's use $K = 40$.

Step 2: Evaluate the ***phase margin*** of the ***uncompensated system.***

Using MATLAB, the phase margin of this (the ***uncompensated*** system) is $PM \cong 18.4$ (deg). This is obviously well below the desired phase margin. See plot below.



Phase Margin of Uncompensated System is 18.4 (deg)

Step 3: Calculate the ratio $\alpha = p/z$

An additional 27 degrees of phase margin are required, so we can calculate

$$\alpha = \frac{1+\sin(\phi_m)}{1-\sin(\phi_m)} = \frac{1+\sin(27)}{1-\sin(27)} = 2.663$$

Step 4: Find ω_m on the Bode diagram of the uncompensated system

The magnitude $-10\log(\alpha) = -4.254$ (db) occurs at approximately 8 (rad/sec). (See plot above.) So, set $\omega_m \cong 8$ (rad/sec), $p = \omega_m\sqrt{\alpha} \approx 13.06$, and $z = p/\alpha \approx 4.9$. In our first iteration,

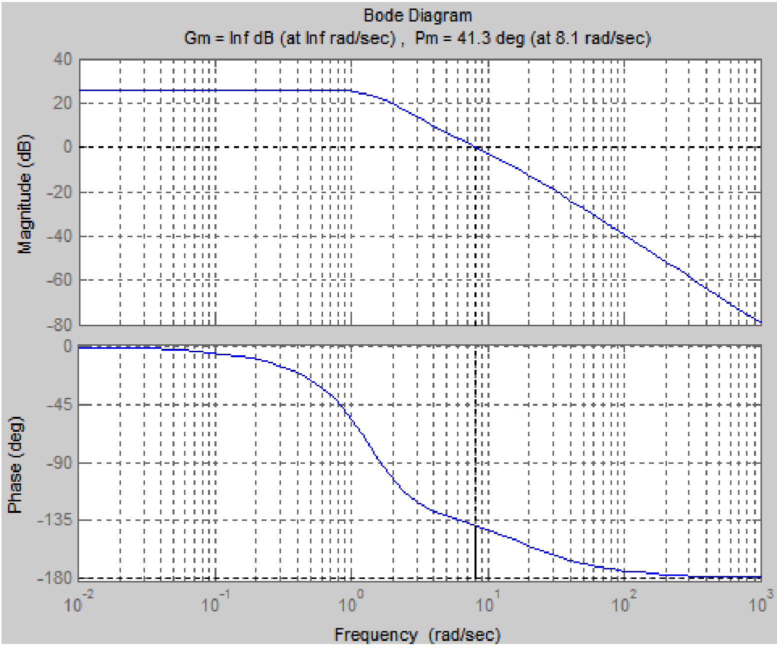
$$G_c(s) = 2.663\left[\frac{s+4.9}{s+13.06}\right]$$ (phase-lead compensator)

Step 5: Check the ***phase margin*** of the ***new compensated system.***

The Bode diagram of the loop transfer function of the compensated system

$$GH(s) = 2.663\left[\frac{s+4.9}{s+13.06}\right]\left[\frac{40}{s^2+2s+2}\right]$$

shows that the phase margin is $PM = 41.3$ (deg). See plot below.



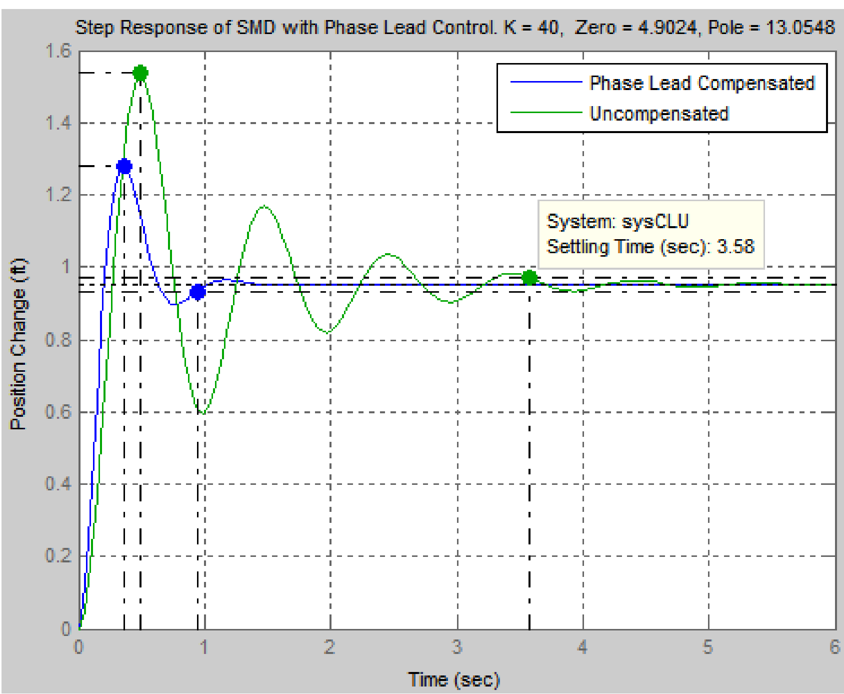
Phase Margin of Compensated System is 41.3 (deg)

Step 6: Repeat steps 3-5 until the desired phase margin is obtained.

Even though the result is below our target, we will use the compensator found above.

Step 7: Check the step response.

Step response of the uncompensated system shows a large overshoot (over 60%) and low damping (settling time of 3.58 (sec)), while the step response of the compensated system shows a smaller overshoot (around 35%) with higher damping (settling time of 0.94 (sec)).



Step response of the compensated system is much better than the uncompensated system. ***Try increasing the phase margin to get a better response.***

Settling times:
Uncompensated = 3.58 (sec)
Compensated = 0.94 (sec)

Frequency Response of Compensated Closed Loop System:

The frequency response of the closed loop system is shown below. The ***bandwidth*** of the closed loop system is approximately 13 (rad/s).