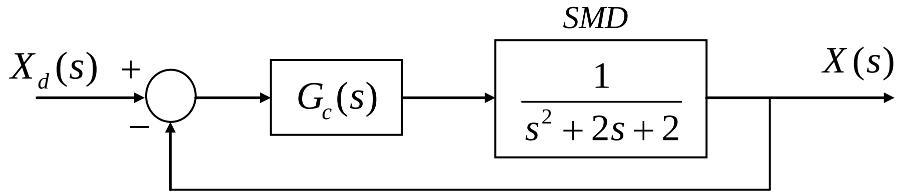
ME 4710 Motion and Control
Root Locus Design of a Phase-Lead Compensator for a Spring-Mass-Damper (SMD) Positioning System

- To illustrate the ***root locus design of a phase-lead compensator***, consider the following SMD positioning system controlled by the compensator $G_c(s)$. Here, $X_d(s)$ and $X(s)$ are the desired and actual positions of the mass.



- Proportional control ($G_c(s)=K$): ***Large gain*** is required to control ***steady-state error*** to a step input. Unfortunately, large gains produce ***undesirable, oscillatory*** closed-loop response.
- Below, we design a phase-lead compensator to ***control*** the ***steady-state error*** and give ***desirable transient response***.

<u>Problem</u>: Design a phase-lead compensator so the closed-loop system has a settling time $T_s < 1$ (sec), a damping ratio of the complex roots $\zeta > 0.5$, and a small steady-state position error. Plot the step response of the resulting closed-loop system.

<u>Root Locus Design</u>: <u>M-file</u>: PhaseLeadPositionControlSMDwithRL.m

Step 1: Examine ***RL diagram*** of ***uncompensated system***. (same as proportional control)

The root locus of the uncompensated system is very simple. The poles of $GH(s)$ are $-1 \pm 1j$. For $K > 0$ the roots move to infinity along the asymptotes at $\sigma_A = -1$. For $K < 0$ the roots move into the break point at –1 and then move along the positive and negative real axis.

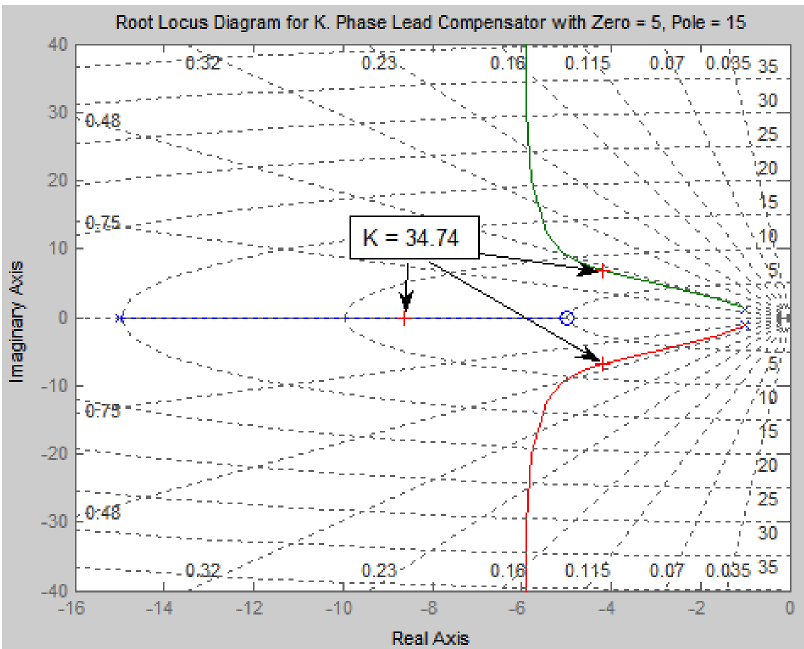
Step 2: Evaluate how the compensator ***changes the RL diagram***.

$GH(s) = \dfrac{K(s+z)}{(s+p)(s^2+2s+2)}$, so the system has asymptotes at $\phi_A = \pm 90$ (deg) that intersect the real axis at $\sigma_A = \dfrac{2(-1)-p+z}{2}$. ***To ensure a settling time of less than 1 second***, we must have $\sigma_A < -4$, or a pole-zero separation of $z - p < -6$. ***Let's assume that*** $z - p = -10$. Note that this is only a starting point. More separation may be necessary.

Step 3: Try ***different pole-zero combinations*** to see effect on ***RL diagram***

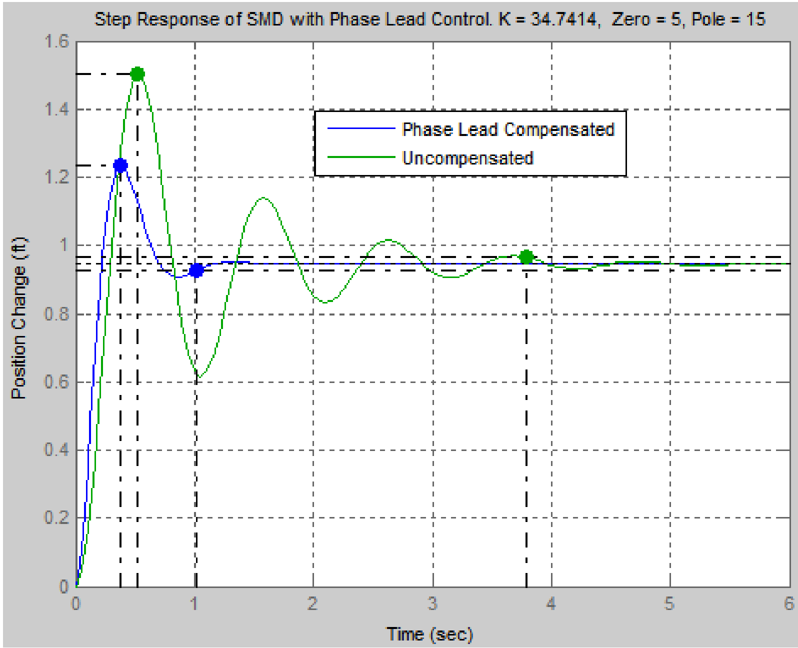
Try $z = 5$ and $p = 15$, so the compensated system is $GH(s) = \dfrac{3K(s+5)}{(s+15)(s^2+2s+2)}$.

- ***Two questions must now be answered***. Can we find roots with $\zeta > 0.5$? Can we use a large enough value for K so that the steady-state error is small? We are lucky with the choice we made. The answer to both questions is "yes!"
- If the answer to either question is "no", then slide the pole-zero combination along the axis in an attempt to satisfy both requirements. Larger pole-zero separations can also be tried.
- The root locus plot below is for the ***compensated system*** and the chosen pole locations correspond to $K \approx 34.74$. Note that the compensator zero (also a zero of the closed loop system) is close to the complex poles, so we expect it to cause larger overshoots than would be expected for $\zeta = 0.5$.



Step 4: Check the step response.

The step response of the ***uncompensated system*** with gain $K \approx 34.74$ (to give the same steady-state error as the compensated system) ***shows a large overshoot*** ($\%OS \approx 59\%$) and ***low damping*** ($T_s \approx 3.8$ (sec)), while the step response of the ***compensated system*** shows a ***smaller overshoot*** ($\%OS \approx 30\%$) with ***higher damping*** ($T_s \approx 1$ (sec)).



- The plots below show the root locus and step response plots for a phase lead compensator with $z = 3$ and $p = 25$. In this case, the compensated system has a 14% overshoot and a settling time of 0.7 seconds.