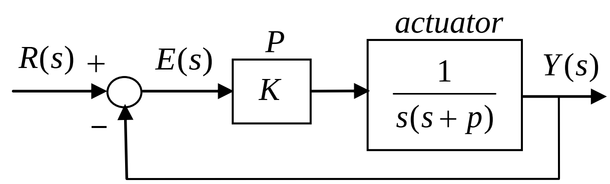
ME 4710 Motion and Control
PID Control of a Hydraulic Actuator: Root Locus Analysis

<u>Proportional (P) Control</u> (ME471HydraulicActuatorPControlwithRL.m)

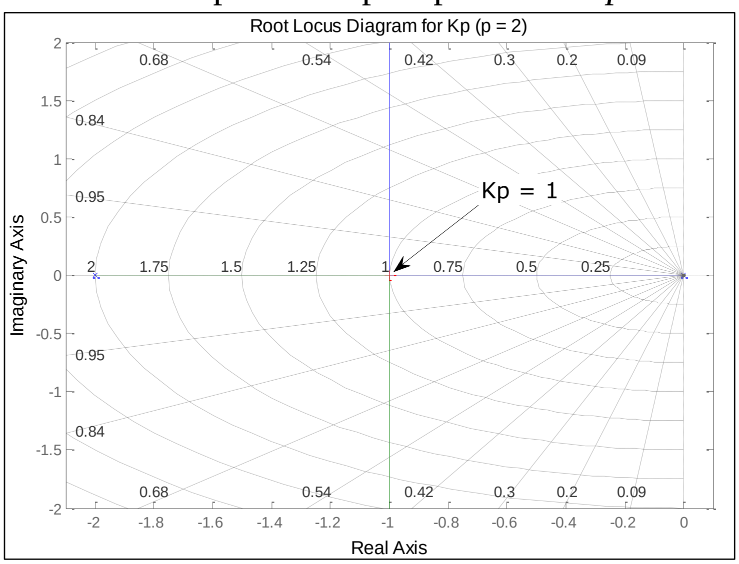
- The diagram shows proportional control of a simple hydraulic actuator. The system has ***two parameters*** – K and p. K is the proportional gain, and p is an actuator parameter that represents how quickly it responds.

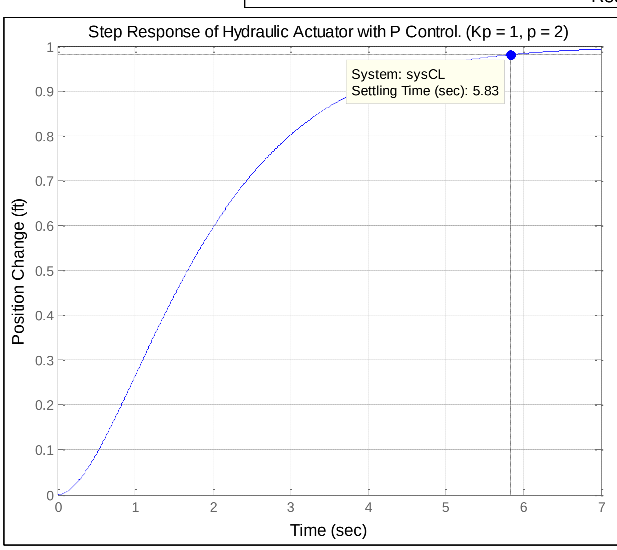


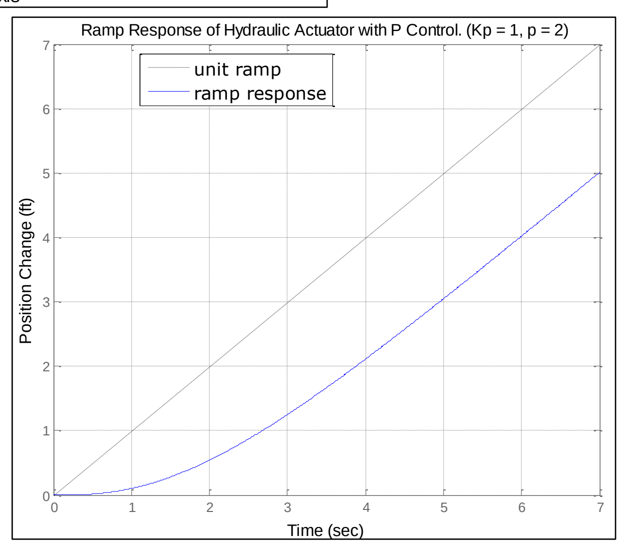
- This is a ***type 1*** system. It has finite error for a ***ramp input***. If $K \geq p$, then $e_{ss} \leq 1$.

$$e_{ss} = \lim_{s\to 0}\left(s \cdot \frac{1}{s^2} \cdot \frac{E}{R}\right) = \lim_{s\to 0}\left(\cancel{s} \cdot \frac{1}{\cancel{s^2}} \cdot \frac{\cancel{s}(s+p)}{s^2+ps+K}\right) = \frac{p}{K} \qquad \text{(ramp input)}$$

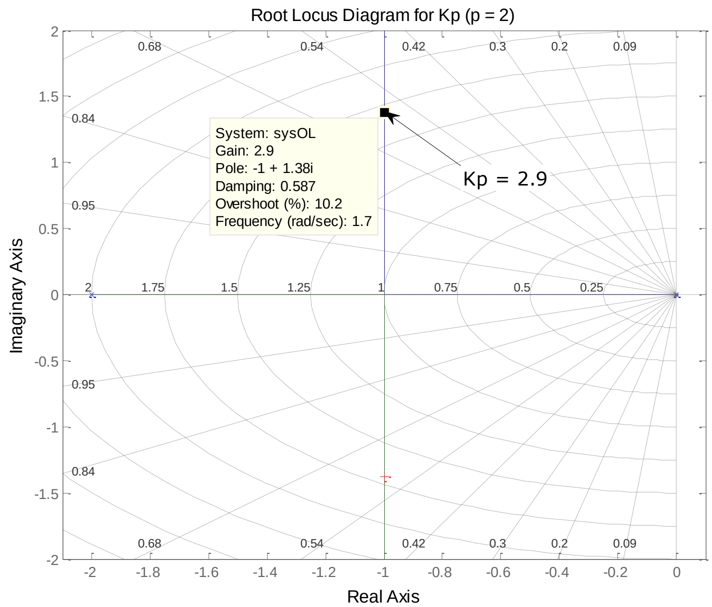
- The RL diagram for K and step and ramp responses for $p = 2$ and $K = 1$:

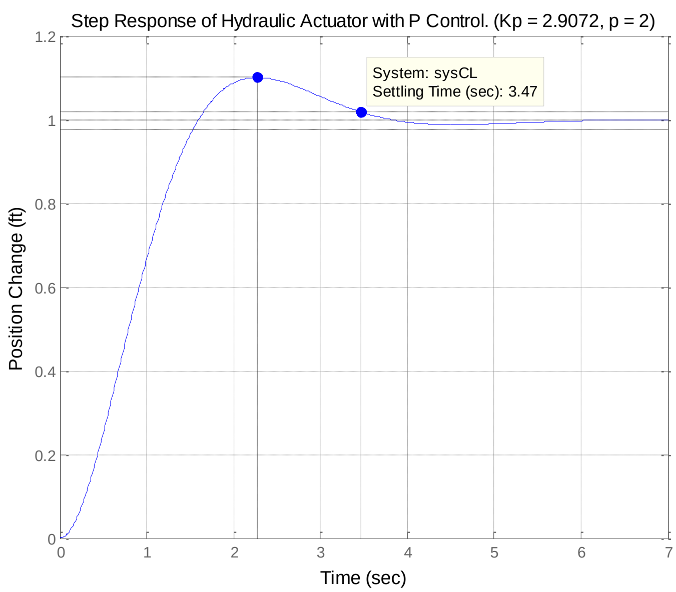


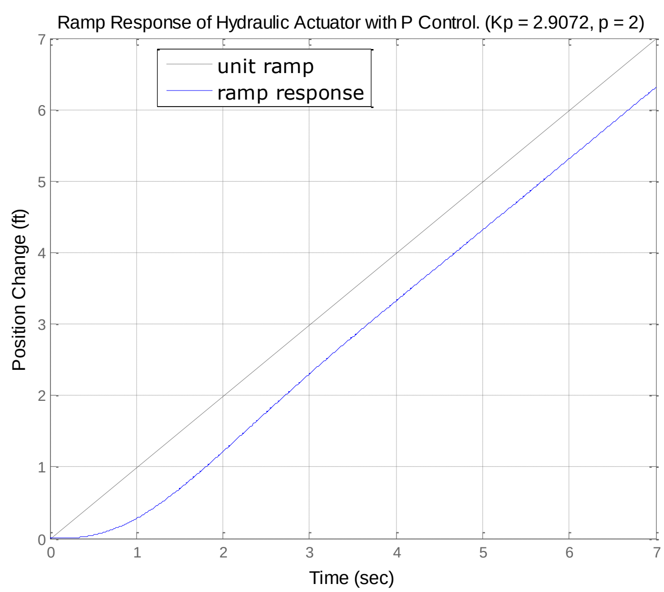




○ The RL diagram for K and step and ramp responses for $p = 2$ and $K = 2.9$:

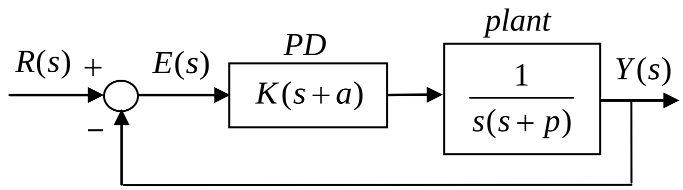






<u>Proportional/Derivative Control</u> (ME471HydraulicActuatorPDControlwithRL.m)

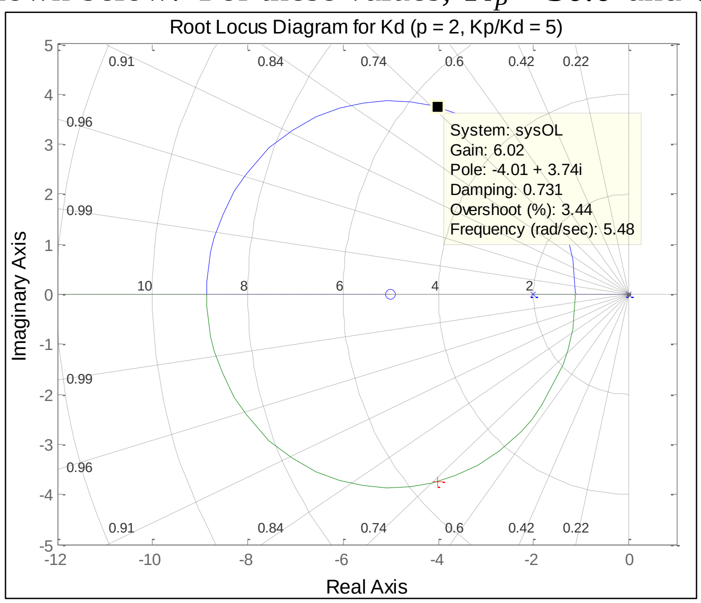
- The system is now shown with *PD* control. Here, $K = K_D$ is the derivative gain, and $a = K_P / K_D$ is the ***ratio*** of the proportional and derivative gains.

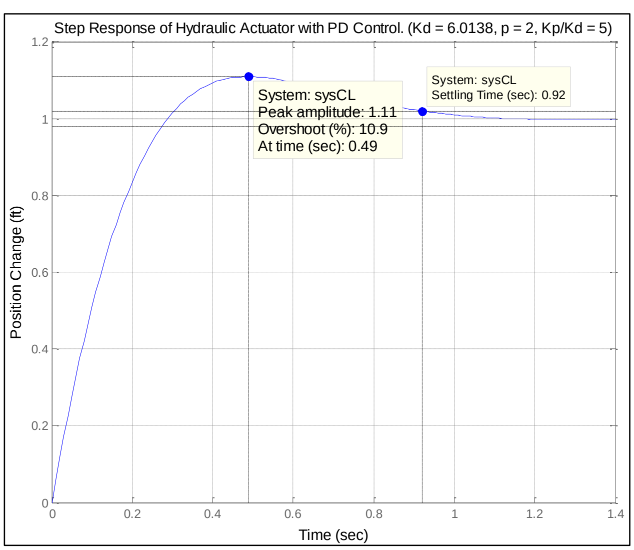


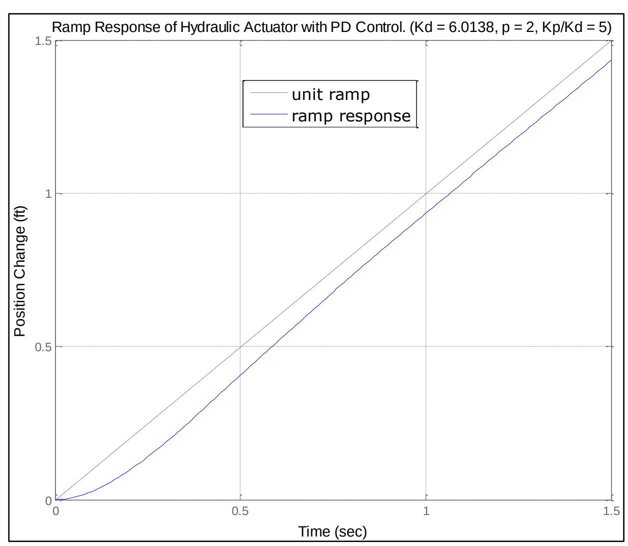
- This is a ***type 1*** system, so it has a finite error for a ramp input. For $e_{ss} \le 1$, we require $aK = K_P \ge p$.

$$e_{ss} = \lim_{s\to 0}\left(s \cdot \frac{1}{s^2} \cdot \frac{E}{R} \right) = \lim_{s\to 0}\left(\not{s} \cdot \frac{1}{\not{s}^{\not{2}}} \cdot \frac{\not{s}(s+p)}{s^2 + (K+p)s + aK} \right) = \frac{p}{aK} = \frac{p}{K_P} \qquad \text{(ramp input)}$$

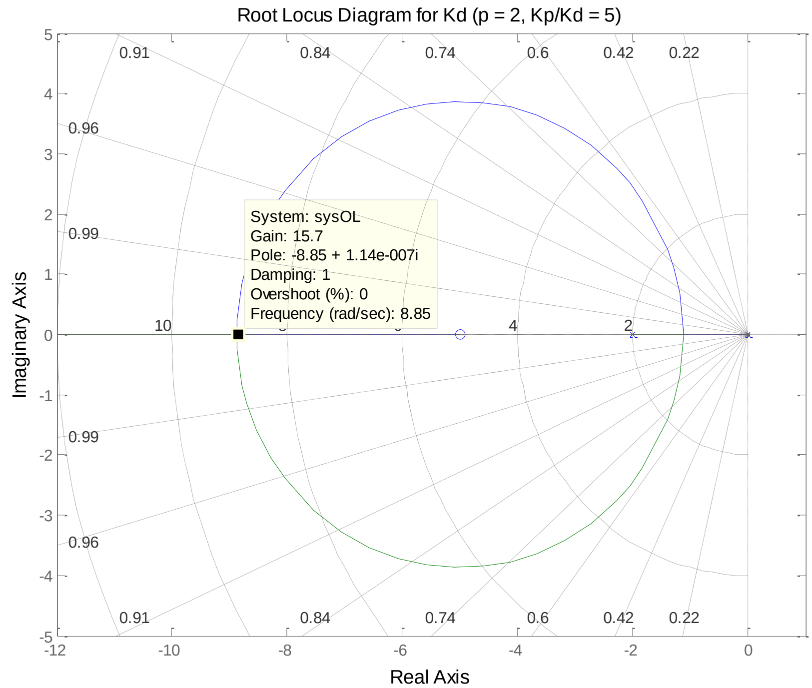
- The RL diagram for parameter K with $p = 2$ and $a = 5$, and step and ramp responses for $K \approx 6.0$ are shown below. For these values, $K_P \approx 30.0$ and $e_{ss} \approx 0.07$.

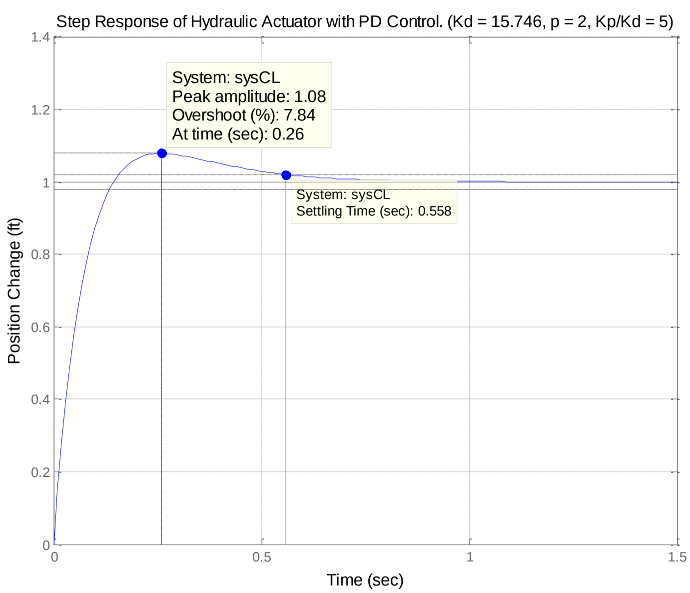


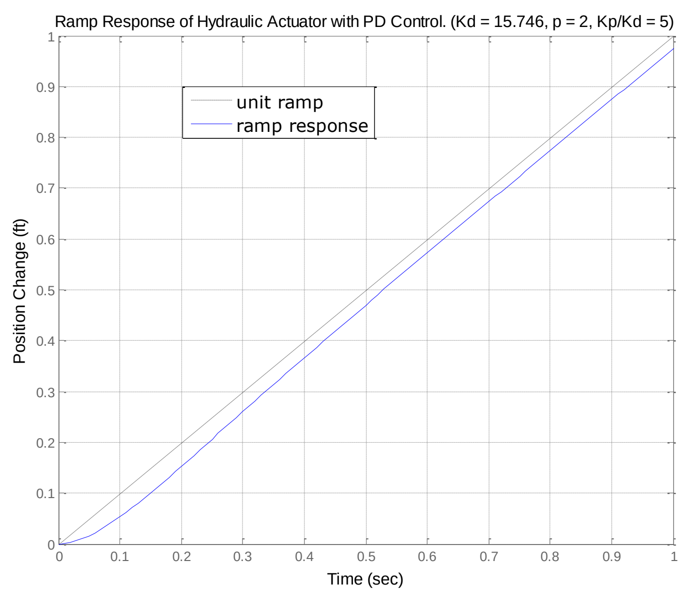




- The RL diagram for parameter K with $p = 2$ and $a = 5$, and step and ramp responses for $K \approx 15.7$ are shown below. For these values, $K_P \approx 78.5$ and $e_{ss} \approx 0.0255$.

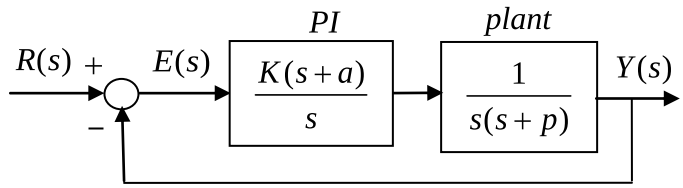




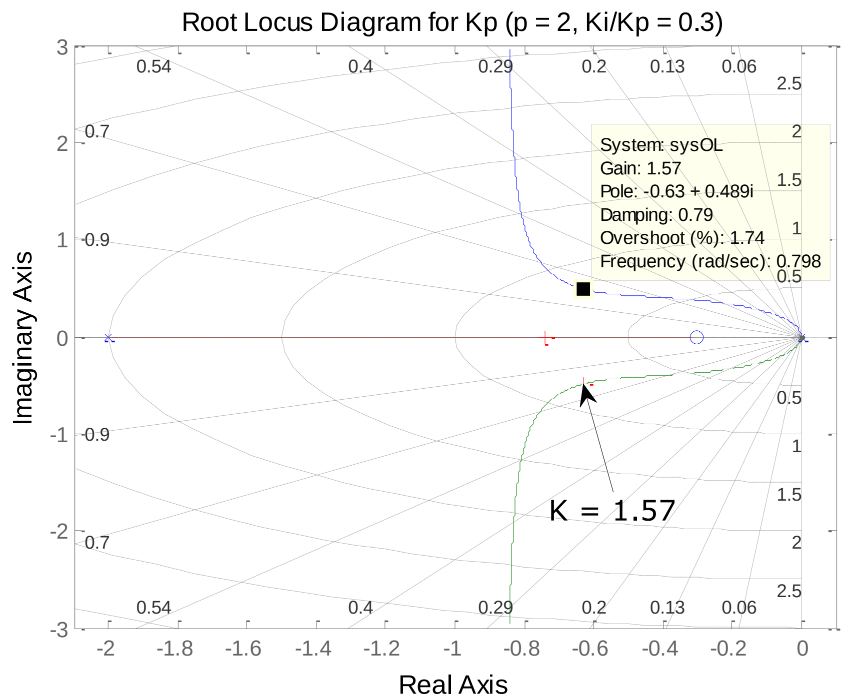


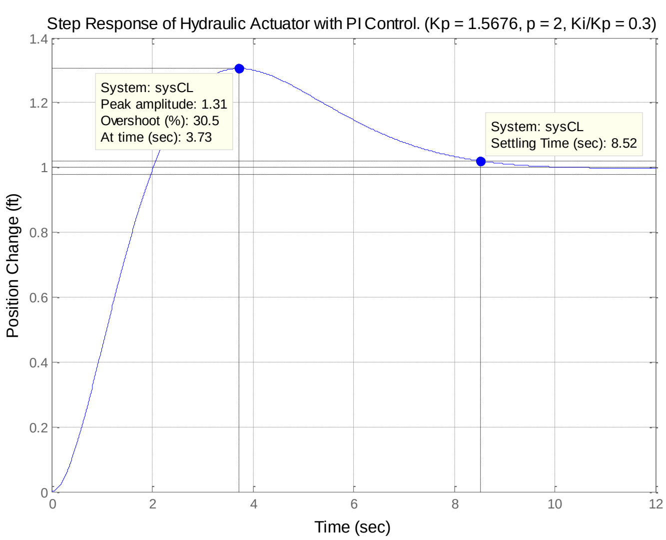
Proportional/Integral Control (ME471HydraulicActuatorPIControlwithRL.m)

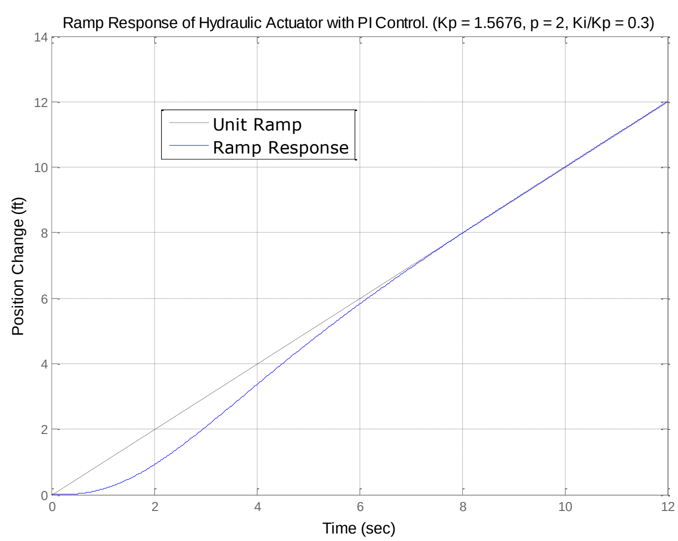
- The system from above is shown here with *PI* control. Here, $K = K_P$ is the proportional gain, and $a = K_I / K_P$ is the ratio of the integral and proportional gains.



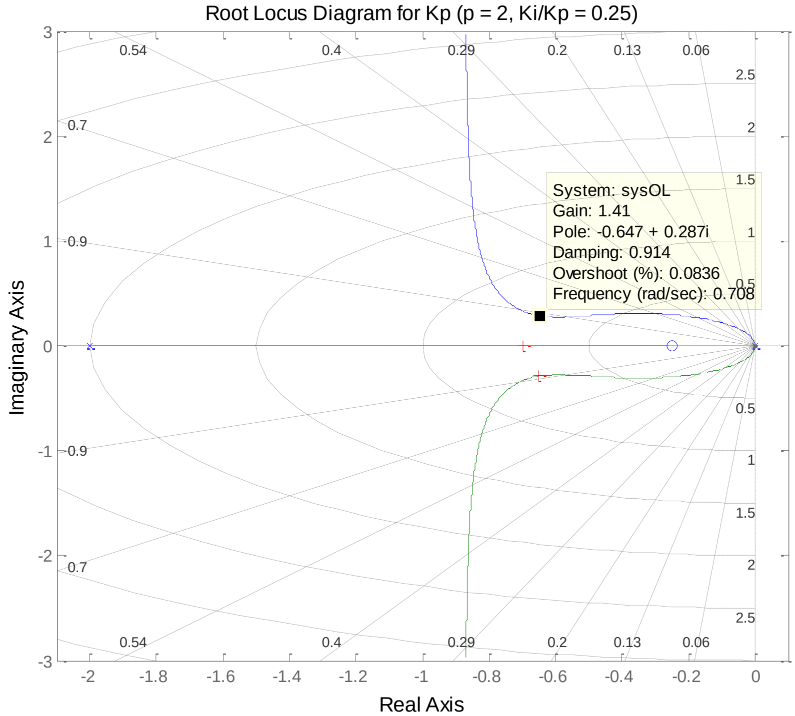
- This is a ***type 2*** system and has zero steady-state error to ***both*** step and ramp inputs.
- The RL diagram for the parameter K with $p = 2$ and $a = 0.3$, and the step and ramp responses for $K \approx 1.57$.

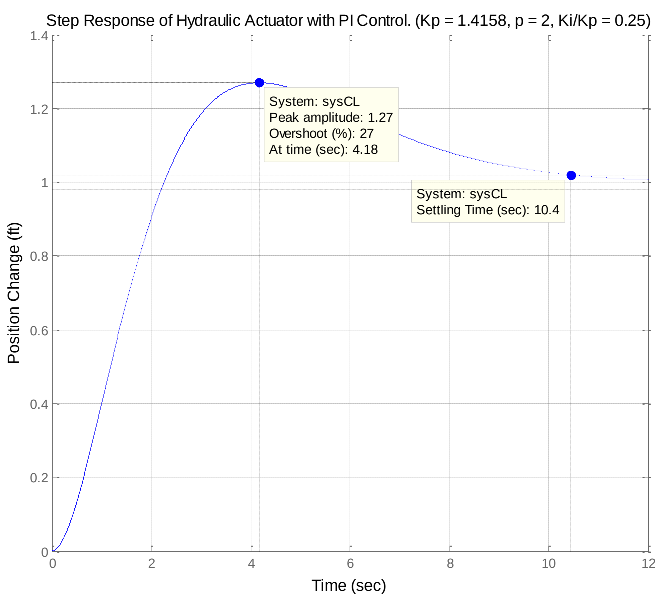


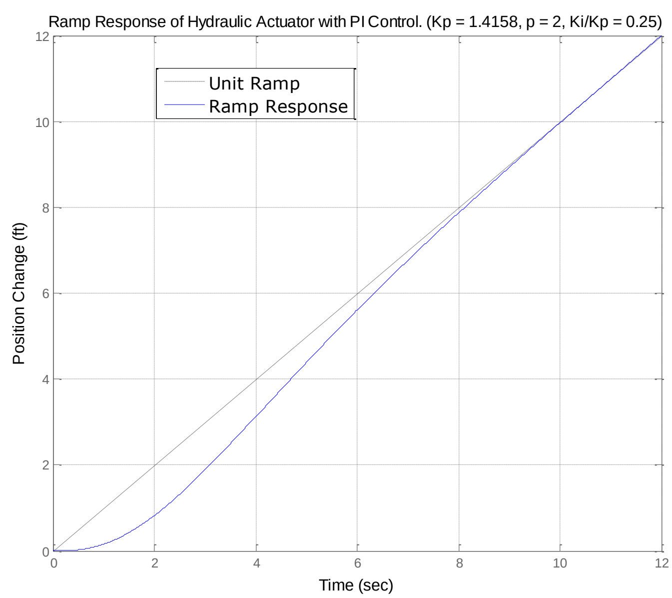




- The RL diagram for the parameter K with $p = 2$ and $a = 0.25$, and the step and ramp responses for $K \approx 1.41$.

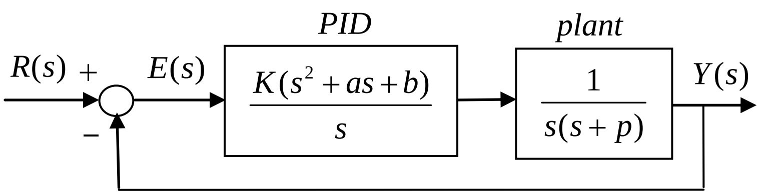






Proportional/Integral/Derivative Control (ME471HydraulicActuatorPIDControlwithRL.m)

- The system from above is shown here with *PID* control. Here, $K = K_D$ is the derivative gain, $a = K_P / K_D$ is the ratio of the proportional and derivative gains, and $b = K_I / K_D$ is the ratio of the integral and derivative gains.



- This is a ***type 2*** system and has zero steady-state error to ***both*** step and ramp inputs.

- The RL diagram for the parameter $K = K_D$ with $p = 2$, $a = K_P / K_D = 4$, and $b = K_I / K_D = 3$, and the step and ramp responses for $K = K_D \approx 9.36$: