

ME 6590 Multibody Dynamics
Angular Acceleration Using Absolute Coordinates

Angle Derivatives as Generalized Speeds: 1-2-3 Rotation Sequence

Recall that the fixed-frame and body-frame components of ${}^R\omega_B$ the angular velocity of a body may be written in matrix-vector form as

$$\boxed{\begin{Bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & S_2 \\ 0 & C_1 & -S_1C_2 \\ 0 & S_1 & C_1C_2 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{Bmatrix} = [\omega_{B,\dot{\theta}}] \{\dot{\theta}\}}$$

"Fixed-frame Components" (1)

$$\boxed{\begin{Bmatrix} \omega'_1 \\ \omega'_2 \\ \omega'_3 \end{Bmatrix} = \begin{bmatrix} C_2C_3 & S_3 & 0 \\ -C_2S_3 & C_3 & 0 \\ S_2 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{Bmatrix} = [\omega'_{B,\dot{\theta}}] \{\dot{\theta}\}}$$

"Body-frame Components" (2)

Consequently, the fixed-frame and body-frame components of ${}^R\alpha_B$ the angular acceleration of the body may be written in the following matrix-vector forms. Recall that the angular velocity vector can be differentiated in either the fixed-frame or the body-frame to find the angular acceleration.

$$\boxed{\begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix} = \begin{Bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{Bmatrix} = [\omega_{B,\dot{\theta}}] \{\ddot{\theta}\} + [\dot{\omega}_{B,\dot{\theta}}] \{\dot{\theta}\}}$$

"Fixed-frame Components" (3)

$$\boxed{\begin{Bmatrix} \alpha'_1 \\ \alpha'_2 \\ \alpha'_3 \end{Bmatrix} = \begin{Bmatrix} \dot{\omega}'_1 \\ \dot{\omega}'_2 \\ \dot{\omega}'_3 \end{Bmatrix} = [\omega'_{B,\dot{\theta}}] \{\ddot{\theta}\} + [\dot{\omega}'_{B,\dot{\theta}}] \{\dot{\theta}\}}$$

"Body-frame Components" (4)

The time derivatives of the partial angular velocity matrices are

$$[\dot{\omega}_{B,\dot{\theta}}] = \begin{bmatrix} 0 & 0 & \dot{\theta}_2 C_2 \\ 0 & -\dot{\theta}_1 S_1 & (S_1 S_2 \dot{\theta}_2 - C_1 C_2 \dot{\theta}_1) \\ 0 & \dot{\theta}_1 C_1 & -(S_1 C_2 \dot{\theta}_1 + C_1 S_2 \dot{\theta}_2) \end{bmatrix} \quad [\dot{\omega}'_{B,\dot{\theta}}] = \begin{bmatrix} -(S_2 C_3 \dot{\theta}_2 + C_2 S_3 \dot{\theta}_3) & C_3 \dot{\theta}_3 & 0 \\ (S_2 S_3 \dot{\theta}_2 - C_2 C_3 \dot{\theta}_3) & -S_3 \dot{\theta}_3 & 0 \\ C_2 \dot{\theta}_2 & 0 & 0 \end{bmatrix}$$

Angular Velocity Components as Generalized Speeds

If the angular velocity components are used as generalized speeds, then we have a much simpler form of the angular acceleration. Recalling that the partial velocity matrices associated with the angular velocity components are 3×3 identity matrices,

$$\left\{ \begin{array}{l} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{array} \right\} = [\omega_{B,\omega}] \{\dot{\omega}\} + [\dot{\omega}_{B,\omega}] \{\omega\} = [\omega_{B,\omega}] \{\dot{\omega}\} = \{\dot{\omega}\} \quad \text{"Fixed-frame Components"}$$

$$\left\{ \begin{array}{l} \alpha'_1 \\ \alpha'_2 \\ \alpha'_3 \end{array} \right\} = [\omega'_{B,\omega'}] \{\dot{\omega}'\} + [\dot{\omega}'_{B,\omega'}] \{\omega'\} = [\omega'_{B,\omega'}] \{\dot{\omega}'\} = \{\dot{\omega}'\} \quad \text{"Body-frame Components"}$$