

ME 6590 Multibody Dynamics

Conversion of Direction Cosines to 1-2-3 Body-Fixed Angle Sequence

Given the coordinate transformation matrix $[R]$, the *orientation angles* for a 1-2-3 body-fixed angle sequence may be computed as follows. First, recall that $[R]$ may be written as

$$[R] = \begin{bmatrix} C_2 C_3 & C_1 S_3 + S_1 S_2 C_3 & S_1 S_3 - C_1 S_2 C_3 \\ -C_2 S_3 & C_1 C_3 - S_1 S_2 S_3 & S_1 C_3 + C_1 S_2 S_3 \\ S_2 & -S_1 C_2 & C_1 C_2 \end{bmatrix}$$

The three orientation angles may then be calculated by observation

$$\theta_2 = \sin^{-1}(R_{31}) \quad (1)$$

$$\theta_1 = \tan^{-1}\left(\frac{-R_{32}}{R_{33}}\right) \quad (2)$$

$$\theta_3 = \tan^{-1}\left(\frac{-R_{21}}{R_{11}}\right) \quad (3)$$

Note that Eqs. (2) and (3) are singular when $\cos(\theta_2) = 0$, that is, when $\theta_2 = \frac{\pi}{2}$. In this case,

$$R_{12} = C_1 S_3 + S_1 C_3 = \sin(\theta_1 + \theta_3) \quad (4)$$

$$R_{22} = C_1 C_3 - S_1 S_3 = \cos(\theta_1 + \theta_3) \quad (5)$$

So, when $\theta_2 = \frac{\pi}{2}$, Eqs. (4) and (5) can be used to *solve for the sum* of the other two angles.

Using Eqs. (4) and (5), write

$$\theta_1 + \theta_3 = \tan^{-1}\left(\frac{R_{12}}{R_{22}}\right)$$