

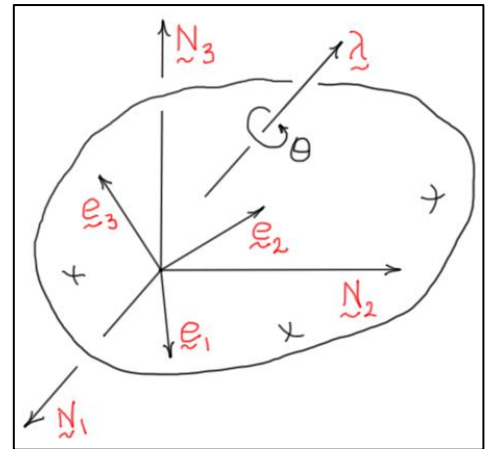
## ME 6590 Multibody Dynamics

### Orientation of a Rigid Body Using Euler Parameters

(Reference: T.R. Kane, P.W. Likins, D.A. Levinson, *Spacecraft Dynamics*, McGraw-Hill, 1983.)

#### Euler's Theorem on Rotation

Consider the rigid body shown in the figure at the right. Let  $R:(\underline{N}_1, \underline{N}_2, \underline{N}_3)$  represent the *base reference frame* and  $B:(\underline{e}_1, \underline{e}_2, \underline{e}_3)$  represent the *body-fixed reference frame* and assume initially that the two frames are *aligned*. Then, *Euler's Theorem on Rotation* states that the rigid body  $B:(\underline{e}_1, \underline{e}_2, \underline{e}_3)$  can be moved into any *arbitrary orientation* relative to the base frame by a rotation about a single axis. Here,  $\theta$  represents the angle of rotation, and the *unit vector*  $\underline{\lambda}$  represents the direction (or axis) of rotation.



#### Euler Parameters

The unit vector  $\underline{\lambda}$  and the angle  $\theta$  can be related to a set of *four parameters* called the Euler *parameters*. First, let  $\underline{\lambda}$  be expressed in terms of the base-frame unit vectors as

$$\underline{\lambda} = \lambda_1 \underline{N}_1 + \lambda_2 \underline{N}_2 + \lambda_3 \underline{N}_3$$

Then, the four Euler parameters may be defined as

$$\varepsilon_i = \lambda_i \sin(\theta/2) \quad (i=1,2,3) \quad \text{and} \quad \varepsilon_4 = \cos(\theta/2)$$

#### Notes

1. The Euler parameters are *not independent*, because  $\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2 = 1$ .
2. It can be shown that

$$\begin{Bmatrix} \underline{e}_1 \\ \underline{e}_2 \\ \underline{e}_3 \end{Bmatrix} = [R] \begin{Bmatrix} \underline{N}_1 \\ \underline{N}_2 \\ \underline{N}_3 \end{Bmatrix} = \begin{bmatrix} (\varepsilon_1^2 - \varepsilon_2^2 - \varepsilon_3^2 + \varepsilon_4^2) & 2(\varepsilon_1 \varepsilon_2 + \varepsilon_3 \varepsilon_4) & 2(\varepsilon_1 \varepsilon_3 - \varepsilon_2 \varepsilon_4) \\ 2(\varepsilon_1 \varepsilon_2 - \varepsilon_3 \varepsilon_4) & (-\varepsilon_1^2 + \varepsilon_2^2 - \varepsilon_3^2 + \varepsilon_4^2) & 2(\varepsilon_2 \varepsilon_3 + \varepsilon_1 \varepsilon_4) \\ 2(\varepsilon_1 \varepsilon_3 + \varepsilon_2 \varepsilon_4) & 2(\varepsilon_2 \varepsilon_3 - \varepsilon_1 \varepsilon_4) & (-\varepsilon_1^2 - \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2) \end{bmatrix} \begin{Bmatrix} \underline{N}_1 \\ \underline{N}_2 \\ \underline{N}_3 \end{Bmatrix}$$

3. *No singularities* exist in these kinematic equations, so many computer programs use Euler parameters to avoid computational singularities; however, they may communicate with the “user” using orientation angles which are easier to *visualize*.