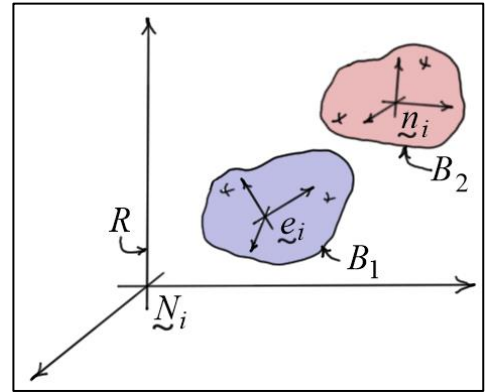


ME 6590 Multibody Dynamics

Coordinate Transformation Matrices Using Relative Coordinates

Instead of describing the orientation of a body relative to the fixed-frame frame, it may be more convenient to describe its orientation *relative to another body* in the system. In such a situation, coordinate transformation matrices can be defined from each body to the inertial frame and from one body to another.

Consider the *two-body system* shown with three reference frames: $R:(\underline{N}_1, \underline{N}_2, \underline{N}_3)$, $B_1:(\underline{e}_1, \underline{e}_2, \underline{e}_3)$, and $B_2:(\underline{n}_1, \underline{n}_2, \underline{n}_3)$. Using these frames, define *four* coordinate *transformation matrices*, $[R_{B_1}]$, $[R_{B_2}]$, $[{}^{B_1}R_{B_2}]$, and $[{}^{B_2}R_{B_1}]$ by the following equations.



$$\boxed{\{\underline{e}\} = [R_{B_1}]\{\underline{N}\}} \quad \boxed{\{\underline{n}\} = [R_{B_2}]\{\underline{N}\}} \quad \boxed{\{\underline{n}\} = [{}^{B_1}R_{B_2}]\{\underline{e}\}} \quad \boxed{\{\underline{e}\} = [{}^{B_2}R_{B_1}]\{\underline{n}\}}$$

From the last two equations, and recalling these matrices are orthogonal, it is clear that

$[{}^{B_1}R_{B_2}]^T = [{}^{B_2}R_{B_1}]$. Also, using the first three equations, it can be written

$$\{\underline{n}\} = [R_{B_2}]\{\underline{N}\} = [{}^{B_1}R_{B_2}]\{\underline{e}\} = [{}^{B_1}R_{B_2}][R_{B_1}]\{\underline{N}\}$$

So,

$$\boxed{[R_{B_2}] = [{}^{B_1}R_{B_2}][R_{B_1}]}$$

This result is *easily extended* to include as *many bodies* as necessary to move from a body frame to a fixed frame *through* frames of a *series of interconnected bodies* of the system.

$$\boxed{[R_{B_i}] = [{}^{B_{i-1}}R_{B_i}][{}^{B_{i-2}}R_{B_{i-1}}] \cdots [{}^{B_1}R_{B_2}][R_{B_1}]}$$

Notes:

1. The transformation matrix $[R_{B_i}]$ converts fixed-frame components into frame B_i .
2. The transformation matrix $[{}^{B_j}R_{B_i}]$ converts frame B_j components into frame B_i .