

ME 2580 Example #35: (Rigid Body Kinematics – Relative Acceleration)

Given: $l_1 = l_2 = 0.4$ (m), $\theta_1 = 25$ (deg), $\theta_2 = 60$ (deg)

$$\omega_1 = \dot{\theta}_1 = 10 \text{ (r/s) (CCW)}, \quad \omega_2 = \dot{\theta}_2 = 5 \text{ (r/s) (CCW)}$$

$$\alpha_1 = \dot{\omega}_1 = 20 \text{ (r/s}^2\text{) (CCW)}, \quad \alpha_2 = \dot{\omega}_2 = -15 \text{ (r/s}^2\text{) (CW)}$$

Find: \underline{a}_B

Solution:

Using the relative acceleration equation,

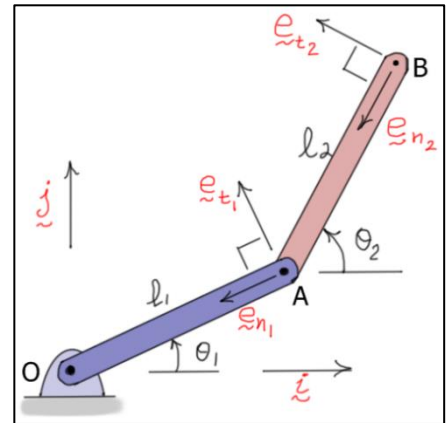
$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$$

where

$$\begin{aligned} \underline{a}_A &= \underline{a}_{A/O} = \left[\underline{\alpha}_1 \times \underline{r}_{A/O} \right] - \left[\omega_1^2 \underline{r}_{A/O} \right] \\ &= \left[20 \underline{k} \times 0.4 \left(\cos(25) \underline{i} + \sin(25) \underline{j} \right) \right] - 10^2 (0.4) \left(\cos(25) \underline{i} + \sin(25) \underline{j} \right) \\ &= 8 \left(-\sin(25) \underline{i} + \cos(25) \underline{j} \right) - 40 \left(\cos(25) \underline{i} + \sin(25) \underline{j} \right) \\ &\approx -39.6333 \underline{i} - 9.65427 \underline{j} \end{aligned}$$

$$\begin{aligned} \underline{a}_{B/A} &= \left[\underline{\alpha}_2 \times \underline{r}_{B/A} \right] - \left[\omega_2^2 \underline{r}_{B/A} \right] \\ &= \left[-15 \underline{k} \times 0.4 \left(\cos(60) \underline{i} + \sin(60) \underline{j} \right) \right] - 5^2 (0.4) \left(\cos(60) \underline{i} + \sin(60) \underline{j} \right) \\ &= -6 \left(-\sin(60) \underline{i} + \cos(60) \underline{j} \right) - 10 \left(\cos(60) \underline{i} + \sin(60) \underline{j} \right) \\ &\approx 0.196152 \underline{i} - 11.6603 \underline{j} \end{aligned}$$

$$\Rightarrow \underline{a}_B \approx -39.4 \underline{i} - 21.3 \underline{j} \text{ (m/s}^2\text{)}$$



Notes:

1. The terms $\underline{\alpha}_1 \times \underline{r}_{A/O}$ and $\underline{\alpha}_2 \times \underline{r}_{B/A}$ are along the \underline{e}_{t_1} and \underline{e}_{t_2} directions, respectively.
2. The terms $[-\omega_1^2 \underline{r}_{A/O}]$ and $[-\omega_2^2 \underline{r}_{B/A}]$ are along the \underline{e}_{n_1} and \underline{e}_{n_2} directions, respectively.