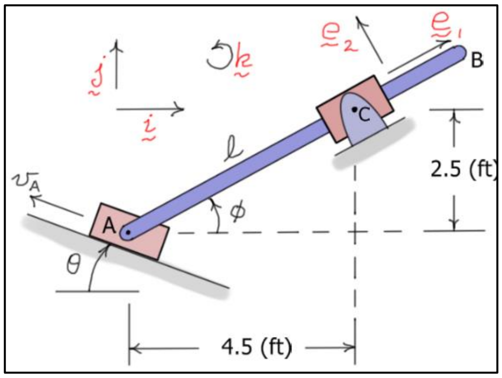
ME 2580 Example #38: (Rigid Body Kinematics – Sliding Contact Acceleration)

Given: $v_A = 2$ (ft/s) = constant , $\theta = 30$ (deg)

Find: a) ω_{AB} , $\dot{\ell}$, b) α_{AB} , $\ddot{\ell}$



Solution #1: using the unit vectors $\left(\underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k}\right)$

a) Using the velocity equation for sliding contacts, write

$$\underset{\sim}{v}_C = \underset{\sim}{v}_A + \left(\underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{C/A}\right) + \underset{\sim}{v}_{C_{\text{rel}}}$$ * (C is fixed, but moves on AB)

where

$$\underset{\sim}{v}_C = \underset{\sim}{0} \qquad \underset{\sim}{v}_A = 2\left(-\cos(20)\,\underset{\sim}{i} + \sin(20)\,\underset{\sim}{j}\right)$$

$$\underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{C/A} = \omega_{AB}\,\underset{\sim}{k} \times \left(4.5\,\underset{\sim}{i} + 2.5\,\underset{\sim}{j}\right) = \omega_{AB}\left(-2.5\,\underset{\sim}{i} + 4.5\,\underset{\sim}{j}\right)$$

$$\underset{\sim}{v}_{C_{\text{rel}}} = \dot{\ell}\,\underset{\sim}{e}_1 = \dot{\ell}\left(4.5\,\underset{\sim}{i} + 2.5\,\underset{\sim}{j}\right)\Big/\ell = \dot{\ell}\left(4.5\,\underset{\sim}{i} + 2.5\,\underset{\sim}{j}\right)\Big/\sqrt{4.5^2 + 2.5^2}$$ (velocity of C on AB)

Substituting into the velocity equation (*) gives the following scalar equations.

$$\begin{aligned} 0 &= -2\cos(20) - 2.5\,\omega_{AB} + \left(\tfrac{4.5}{\ell}\right)\dot{\ell} \\ 0 &= \;\; 2\sin(20) + 4.5\,\omega_{AB} + \left(\tfrac{2.5}{\ell}\right)\dot{\ell} \end{aligned} \quad \ldots\text{solving gives} \Rightarrow \begin{cases} \omega_{AB} \approx -0.293458 \approx -0.293 \text{ (r/s)} \\ \dot{\ell} \approx 1.31068 \approx 1.31 \text{ (ft/s)} \end{cases}$$

b) Using the acceleration equation for sliding contacts, write

$$\underset{\sim}{a}_C = \underset{\sim}{a}_A + \left(\underset{\sim}{\alpha}_{AB} \times \underset{\sim}{r}_{C/A}\right) - \omega_{AB}^2\,\underset{\sim}{r}_{C/A} + \underset{\sim}{a}_{C_{\text{rel}}} + 2\left(\underset{\sim}{\omega}_{AB} \times \underset{\sim}{v}_{C_{\text{rel}}}\right)$$ ** (C is fixed, but moves on AB)

where

$$\underset{\sim}{a}_C = \underset{\sim}{a}_A = \underset{\sim}{0}$$

$$\underset{\sim}{\alpha}_{AB} \times \underset{\sim}{r}_{C/A} = \alpha_{AB}\,\underset{\sim}{k} \times \left(4.5\,\underset{\sim}{i} + 2.5\,\underset{\sim}{j}\right) = \alpha_{AB}\left(-2.5\,\underset{\sim}{i} + 4.5\,\underset{\sim}{j}\right) \qquad -\omega_{AB}^2\,\underset{\sim}{r}_{C/A} = -\omega_{AB}^2\left(4.5\,\underset{\sim}{i} + 2.5\,\underset{\sim}{j}\right)$$

$$\underset{\sim}{a}_{C_{\text{rel}}} = \ddot{\ell}\,\underset{\sim}{e}_1 = \ddot{\ell}\left(4.5\,\underset{\sim}{i} + 2.5\,\underset{\sim}{j}\right)\Big/\ell = \ddot{\ell}\left(4.5\,\underset{\sim}{i} + 2.5\,\underset{\sim}{j}\right)\Big/\sqrt{4.5^2 + 2.5^2}$$ (acceleration of C on AB)

$$2\left(\underset{\sim}{\omega}_{AB} \times \underset{\sim}{v}_{C_{\text{rel}}}\right) = 2\omega_{AB}\,\underset{\sim}{k} \times \dot{\ell}\left(4.5\,\underset{\sim}{i} + 2.5\,\underset{\sim}{j}\right)\Big/\ell = \left(\frac{2\omega_{AB}\dot{\ell}}{\ell}\right)\left(-2.5\,\underset{\sim}{i} + 4.5\,\underset{\sim}{j}\right)$$

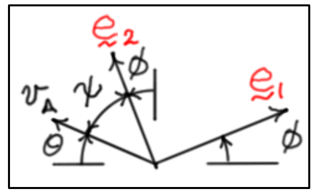
Substituting into the acceleration equation (**) gives the following scalar equations.

$$\begin{aligned} 0 &= -2.5\,\alpha_{AB} - 4.5\,\omega_{AB}^2 + \left(\tfrac{4.5}{\ell}\right)\ddot{\ell} - \left(\frac{5\omega_{AB}\dot{\ell}}{\ell}\right) \\ 0 &= \;\; 4.5\,\alpha_{AB} - 2.5\,\omega_{AB}^2 + \left(\tfrac{2.5}{\ell}\right)\ddot{\ell} + \left(\frac{9\omega_{AB}\dot{\ell}}{\ell}\right) \end{aligned} \quad \ldots\text{solving gives} \Rightarrow \begin{cases} \alpha_{AB} \approx 0.149434 \approx 0.149 \text{ (r/s}^2\text{)} \\ \ddot{\ell} \approx 0.443318 \approx 0.443 \text{ (ft/s}^2\text{)} \end{cases}$$

Conclusions:

1. Bar AB is rotating clockwise $(\omega_{AB} < 0)$, but slowing down $(\alpha_{AB} > 0)$.
2. Point C is moving away from point A $(\dot{\ell} > 0)$ at an increasing rate $(\ddot{\ell} > 0)$.

Solution #2: using the unit vectors $\left(\underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{k} \right)$



a) Using the velocity equation for sliding contacts, write

$$\boxed{\underset{\sim}{v}_C = \underset{\sim}{v}_A + \left(\underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{C/A} \right) + \underset{\sim}{v}_{C_{\text{rel}}}}$$ *** (C is fixed, but moves on AB)

where

$\boxed{\underset{\sim}{v}_C = \underset{\sim}{0}}$ $\boxed{\underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{C/A} = \omega_{AB} \underset{\sim}{k} \times \ell \underset{\sim}{e}_1 = \ell \omega_{AB} \underset{\sim}{e}_2}$ $\boxed{\underset{\sim}{v}_{C_{\text{rel}}} = \dot{\ell} \underset{\sim}{e}_1}$ (velocity of C on AB)

$\boxed{\underset{\sim}{v}_A = 2\left(-\sin(\psi) \underset{\sim}{e}_1 + \cos(\psi) \underset{\sim}{e}_2 \right)}$ $\boxed{\psi = 90 - \phi - \theta \approx 40.9454 \text{ (deg)}}$

Substituting into the velocity equation (***) gives the following scalar equations.

$$\boxed{\begin{aligned} 0 &= -2\sin(\psi) + \dot{\ell} \\ 0 &= \;\; 2\cos(\psi) + \ell \omega_{AB} \end{aligned}} \quad \ldots\text{solving gives} \Rightarrow \boxed{\begin{cases} \omega_{AB} \approx -0.293458 \approx -0.293 \text{ (r/s)} \\ \dot{\ell} \approx 1.31068 \approx 1.31 \text{ (ft/s)} \end{cases}}$$

b) Using the acceleration equation for sliding contacts, write

$$\boxed{\underset{\sim}{a}_C = \underset{\sim}{a}_A + \left(\underset{\sim}{\alpha}_{AB} \times \underset{\sim}{r}_{C/A} \right) - \omega_{AB}^2 \underset{\sim}{r}_{C/A} + \underset{\sim}{a}_{C_{\text{rel}}} + 2\left(\underset{\sim}{\omega}_{AB} \times \underset{\sim}{v}_{C_{\text{rel}}} \right)}$$ **** (C is fixed, but moves on AB)

where

$\boxed{\underset{\sim}{a}_C = \underset{\sim}{a}_A = \underset{\sim}{0}}$ $\boxed{\underset{\sim}{\alpha}_{AB} \times \underset{\sim}{r}_{C/A} = \alpha_{AB} \underset{\sim}{k} \times \ell \underset{\sim}{e}_1 = \ell \alpha_{AB} \underset{\sim}{e}_2}$ $\boxed{-\omega_{AB}^2 \underset{\sim}{r}_{C/A} = -\ell \omega_{AB}^2 \underset{\sim}{e}_1}$

$\boxed{\underset{\sim}{a}_{C_{\text{rel}}} = \ddot{\ell} \underset{\sim}{e}_1}$ (acceleration of C on AB) $\boxed{2\left(\underset{\sim}{\omega}_{AB} \times \underset{\sim}{v}_{C_{\text{rel}}} \right) = 2\omega_{AB} \underset{\sim}{k} \times \dot{\ell} \underset{\sim}{e}_1 = 2\omega_{AB} \dot{\ell} \underset{\sim}{e}_2}$

Substituting into the acceleration equation (****) gives the following scalar equations.

$$\boxed{\begin{aligned} 0 &= -\ell \omega_{AB}^2 + \ddot{\ell} \\ 0 &= \;\; \ell \alpha_{AB} + 2\dot{\ell} \omega_{AB} \end{aligned}} \quad \ldots\text{solving gives} \Rightarrow \boxed{\begin{cases} \alpha_{AB} \approx 0.149434 \approx 0.149 \text{ (r/s}^2\text{)} \\ \ddot{\ell} \approx 0.443318 \approx 0.443 \text{ (ft/s}^2\text{)} \end{cases}}$$

Conclusions:

1. As expected, the results are the ***same*** as above. The results ***do not depend*** on the unit vectors used to express the equations.
2. The unit vectors chosen for the second solution produce a ***less complicated set*** of scalar equations. A ***careful selection*** of unit vectors (directions) can often make the solution process ***less tedious***.