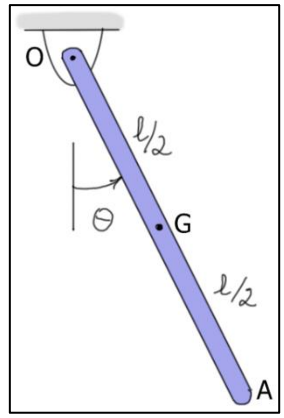
ME 2580 Example #42: (Rigid Body Kinetics – Fixed Axis Rotation)



Given: $W = 10$ (lb), $\ell = 2$ (ft), $\theta = 30$ (deg), $\underset{\sim}{\omega} = 5\underset{\sim}{k}$ (r/s)

Find: $\underset{\sim}{\alpha}$, the angular acceleration of the bar

reaction forces on the pin at O

Solution:

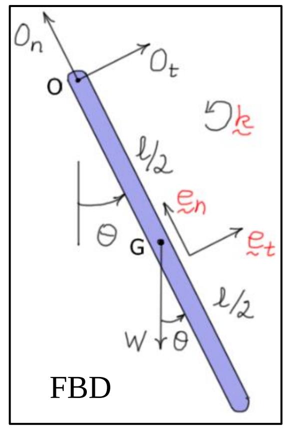
Applying Newton's laws of motion to the free-body-diagram in the normal and tangential directions and taking moments about the fixed-point O, write

$$\sum F_n = O_n - W\cos(\theta) = ma_n = m\left(\tfrac{1}{2}\ell\omega^2\right)$$

$$\Rightarrow\ O_n = 10\cos(30) + \tfrac{1}{2}\tfrac{10}{g}(2)5^2 = 16.4242 \approx 16.4 \text{ (lb)}$$

$$\sum F_t = O_t - W\sin(\theta) = ma_t = m\left(\tfrac{1}{2}\ell\alpha\right)$$

$$\sum M_O = -\left(\tfrac{1}{2}\ell\sin(\theta)\right)W = I_O\alpha \qquad I_O = \tfrac{1}{3}m\ell^2 = \tfrac{1}{3}\tfrac{W}{g}2^2 = \tfrac{4W}{3g}$$



Solving the moment equation for the angular acceleration gives

$$\alpha = -\frac{W\ell\sin(\theta)}{2I_O} = -\frac{(W\ell\sin(\theta))(3g)}{2(4W)} = -\frac{(2\sin(30))(3g)}{8} = -12.075 \approx -12.1 \text{ (r/s}^2\text{)}$$

Substituting this result into the tangential force equation gives

$$O_t = W\sin(\theta) + m\left(\tfrac{1}{2}\ell\alpha\right) = (10\sin(30)) + \tfrac{1}{2}\tfrac{10}{g}(2\alpha) \approx 1.25 \text{ (lb)}$$

Alternatively, the moment equation could be written about the mass-center G. This results in a ***second equation*** for O_t and α that can be solved ***simultaneously*** with the tangential force equation.

$$\sum M_G = -\left(\tfrac{1}{2}\ell\right)O_t = I_G\alpha = \left(\tfrac{1}{12}m\,\ell^2\right)\alpha = \tfrac{1}{12}\left(\tfrac{W}{g}\right)2^2\alpha = \left(\tfrac{W}{3g}\right)\alpha$$

Simultaneous equations:

$$\begin{aligned} O_t - \left(\tfrac{W}{g}\right)\alpha &= W\sin(30) = 5 \\ O_t + \left(\tfrac{W}{3g}\right)\alpha &= 0 \end{aligned} \quad\Rightarrow\quad \begin{cases} O_t \approx 1.25 \text{ (lb)} \\ \alpha \approx -12.1 \text{ (r/s}^2\text{)} \end{cases}$$