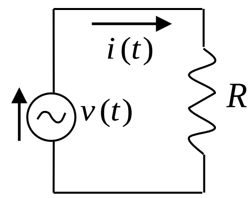
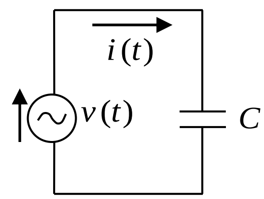
ENGR 1990 Engineering Mathematics
Application of Integration in Electrical Engineering

Current–Voltage Relationships for Resistors, Capacitors, and Inductors



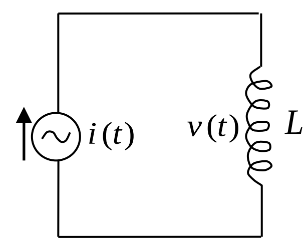
The voltage across and the current through a ***resistor*** are related simply by its resistance.

$$\boxed{v(t) = R\,i(t)} \quad \text{or} \quad \boxed{i(t) = v(t)/R}$$



The voltage across and the current through a ***capacitor*** are related by the equations

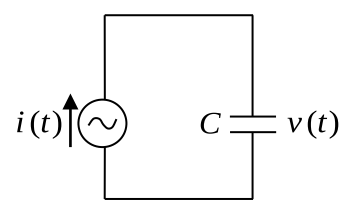
$$\boxed{i(t) = C\frac{dv}{dt}} \quad \text{or} \quad \boxed{v(t) = \tfrac{1}{C}\int i(t)\,dt}$$



The voltage across and the current through an ***inductor*** are related by the equations

$$\boxed{v(t) = L\frac{di}{dt}} \quad \text{or} \quad \boxed{i(t) = \tfrac{1}{L}\int v(t)\,dt}$$

<u>Example 1:</u>



<u>Given:</u> A current $i(t) = 5\left(1 - e^{-3t}\right)$ (amps) is applied to a capacitor with $C = 0.5$ (f).

<u>Find:</u> $v(t)$ the voltage across the capacitor. Assume $v(0) = 0$.

<u>Solution:</u>

The voltage may be found by integrating the current.

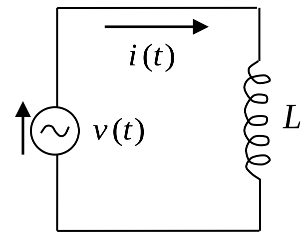
$$v(t) = \tfrac{1}{C}\int i(t)\,dt = 2\int 5\left(1 - e^{-3t}\right)dt = 10\left(t + \tfrac{1}{3}e^{-3t}\right) + D$$

To find the constant D, we apply the ***initial condition***

$$v(0) = 0 = 10\left(\tfrac{1}{3}e^{0}\right) + D \quad \Rightarrow \boxed{D = \tfrac{-10}{3}} \quad \Rightarrow \quad \boxed{v(t) = 10\left(-\tfrac{1}{3} + t + \tfrac{1}{3}e^{-3t}\right) \text{ (volts)}}$$

Check: $\boxed{i(t) = C\frac{dv}{dt} = 0.5\frac{d}{dt}\left[10\left(-\tfrac{1}{3} + t + \tfrac{1}{3}e^{-3t}\right)\right] = 5\left(0 + 1 - e^{-3t}\right) = 5\left(1 - e^{-3t}\right)}$

Example 2:



Given: A voltage $v(t) = 120\sin(60\pi t)$ (volts) is applied to an inductor with $L = 500$ (mh).

Find: the current $i(t)$ through the inductor. Assume $i(0) = 0$.

Solution:

To find the current we integrate the voltage.

$$i(t) = \tfrac{1}{L}\int v(t)\,dt = 2\int 120\sin(60\pi t)\,dt = 240\left[\frac{-\cos(60\pi t)}{60\pi}\right] + D$$

To find the constant *D*, we apply the ***initial condition***

$$i(0) = 0 = D - \tfrac{4}{\pi} = 0 \quad \Rightarrow \boxed{D = \tfrac{4}{\pi}} \quad \Rightarrow \quad \boxed{i(t) = \tfrac{4}{\pi}\left[1 - \cos(60\pi t)\right] \text{ (amps)}}$$

Check: $\boxed{v(t) = L\dfrac{di}{dt} = 0.5\dfrac{d}{dt}\left(\tfrac{4}{\pi}\left(1 - \cos(60\pi t)\right)\right) = \tfrac{2}{\pi}\left(60\pi\sin(60\pi t)\right) = 120\sin(60\pi t)}$