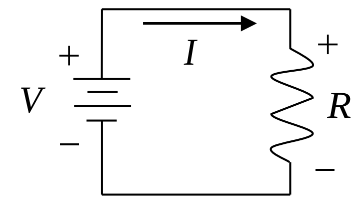
ENGR 1990 Engineering Mathematics
Application of Lines in Electric Circuits

DC Circuit with a Single Resistor



A direct current (DC) circuit with a single resistor is shown in the diagram. The symbol V represents the applied voltage, the symbol R represents the resistance of the resistor, and the symbol I represents the current flowing through the circuit.

Symbol	Description	Units
V	Applied voltage (e.g. battery voltage)	volts (V)
R	Resistance of resistor	ohms (Ω)
I	Current passing through the circuit	amps (A)

There are ***two important laws*** that enable us to relate these three variables, ***Ohm's Law*** and ***Kirchhoff's Voltage Law***.

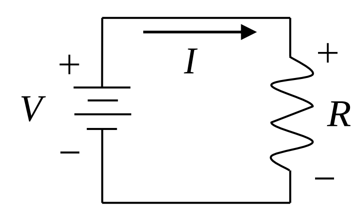
	Equation	Description
Ohm's Law	$V_R = R\,I$	The ***voltage drop across the resistor*** is the product of its resistance and the current passing through it.
Kirchhoff's Voltage Law	$\sum(\text{voltage rises}) = \sum(\text{voltage drops})$	The sum of voltage rises must equal the sum of the voltage drops around any loop in a circuit.

Applying Kirchhoff's voltage law and Ohm's law to the circuit gives $\boxed{V = V_R = R\,I}$. This is the equation of a line relating the current to the applied voltage. The slope (m) of the line is the resistance R, and the intercept (b) is zero.

Example #1

Given: For the circuit shown, the following data were measured.

V (volts)	I (amps)
5	0.05
10	0.1



Find: The resistance R.

Solution:

The current is related to the applied voltage by the equation $\boxed{\underset{y\;=\;m\;x}{V = R\,I}}$, so we can find the resistance by finding the slope of the line.

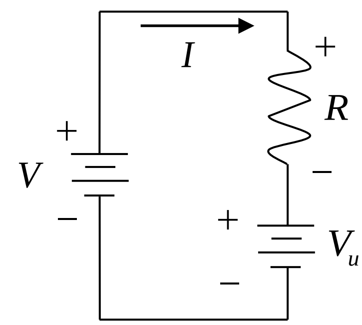
$$\boxed{R = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{V_2 - V_1}{I_2 - I_1} = \frac{10-5}{0.1-0.05} = 100 \text{ (ohms, } \Omega)}$$

So now we have: $\boxed{V = 100\, I}$

Note: Because the intercept is zero, we could have used the point $(x_1, y_1) = (0,0)$ with either of the data values given.

Example #2

Given: In the circuit shown, V is a ***known*** voltage source and V_u is an ***unknown*** constant voltage source. For the circuit shown, the following data were measured.



V (volts)	I (amps)
15	0.03333
30	0.1333

Find: The resistance R and the unknown voltage source V_u.

Solution:

Applying Kirchhoff's voltage law to this circuit gives $\boxed{\underset{y}{V} = V_R + V_u = \underbrace{R\,I + V_u}_{m\,x+b}}$. So, the resistance R is the ***slope*** (as before), and the unknown voltage V_u is the ***intercept***.

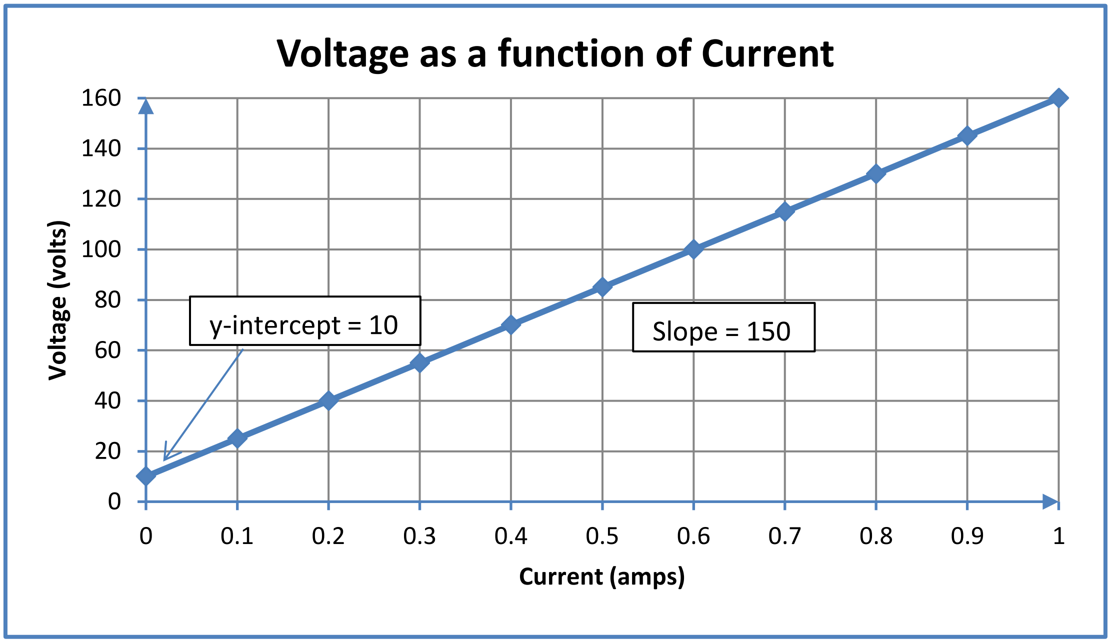
$$\boxed{R = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{V_2 - V_1}{I_2 - I_1} = \frac{30-15}{0.1333-0.03333} = 150.05 \approx 150\ (\Omega)}$$

So now we have: $\boxed{V = 150\, I + V_u}$

To find the intercept, we use this result with ***either*** of the two data points.

$$\boxed{V_u = V - 150\, I = \begin{cases} 15 - (150 \times 0.03333) = 10.0005 \approx 10 \text{ (volts)} \\ \quad \text{or} \\ 30 - (150 \times 0.1333) = 10.005 \approx 10 \text{ (volts)} \end{cases}}$$

We now have the completed equation: $\boxed{V = 150I + 10}$



Example #3

In the above examples, the current I is the ***independent variable*** and the voltage V is the ***dependent variable***. We can easily reverse these roles by solving the completed equation for the current I as a function of the voltage V.

$$V = 150I + 10 \quad \Rightarrow \quad 150I = V - 10 \quad \Rightarrow \quad \boxed{I = \left(\tfrac{1}{150}\right)V - \left(\tfrac{10}{150}\right)}$$