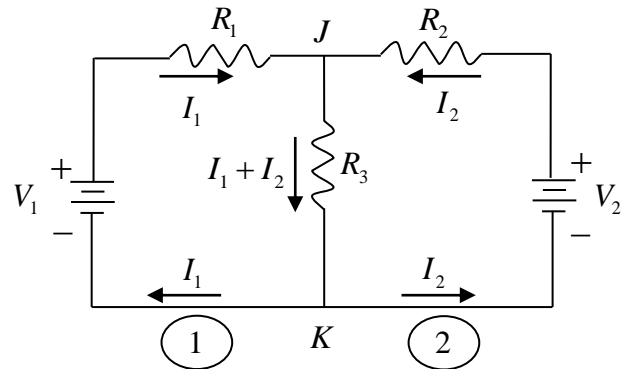


## ENGR 1990 Engineering Mathematics

### Applications of Systems of Linear, Algebraic Equations in Electrical Engineering

Consider the double-loop DC circuit shown in the diagram. Application of **Kirchhoff's Current Law** tells us that the current in the middle path (common to both loops) is the **sum** of the currents flowing in the outer paths. Using this result, we apply **Kirchhoff's Voltage Law** to each loop separately.



$$\sum_{\text{loop 1}} (\text{voltage rises}) = V_1 = \sum_{\text{loop 1}} (\text{voltage drops}) = R_1 I_1 + R_3 (I_1 + I_2)$$

$$\sum_{\text{loop 2}} (\text{voltage rises}) = V_2 = \sum_{\text{loop 2}} (\text{voltage drops}) = R_2 I_2 + R_3 (I_1 + I_2)$$

or

$$\begin{cases} (R_1 + R_3)I_1 + (R_3)I_2 = V_1 \\ (R_3)I_1 + (R_2 + R_3)I_2 = V_2 \end{cases} \quad (1)$$

Given the resistances and the applied voltages, Eqs. (1) can be solved for the two currents.

#### Example

Given:  $R_1 = 6(\Omega)$ ,  $R_2 = 10(\Omega)$ ,  $R_3 = 5(\Omega)$ ,  $V_1 = 6$  (volts), and  $V_2 = 12$  (volts)

Find: the currents  $I_1$  and  $I_2$  using a) substitution, and b) Cramer's rule.

#### Solution:

Using the given values, we have the two simultaneous equations: 
$$\begin{cases} 11I_1 + 5I_2 = 6 \\ 5I_1 + 15I_2 = 12 \end{cases}.$$

#### a) Substitution:

Solving the first equation for  $I_2$  in terms of  $I_1$ , gives 
$$I_2 = \frac{6 - 11I_1}{5}.$$
 Substituting this result into the second equation for  $I_2$  gives a single equation that we can solve for  $I_1$ .

$$12 = 5I_1 + 15\left(\frac{6 - 11I_1}{5}\right) = 5I_1 + 3(6 - 11I_1) = -28I_1 + 18$$

$$\Rightarrow I_1 = \frac{18 - 12}{28} = 0.2143 \text{ (amps)} \quad \text{and} \quad I_2 = \left(\frac{6 - 11I_1}{5}\right) = 0.7286 \text{ (amps)}$$

b) Cramer's Rule:

To use Cramer's rule, we first write the equations as a *matrix equation*.

$$\begin{bmatrix} 11 & 5 \\ 5 & 15 \end{bmatrix} \begin{Bmatrix} I_1 \\ I_2 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 12 \end{Bmatrix}$$

Then, we solve.

$$I_1 = \frac{\det \begin{bmatrix} 6 & 5 \\ 12 & 15 \end{bmatrix}}{\det \begin{bmatrix} 11 & 5 \\ 5 & 15 \end{bmatrix}} = \frac{(6 \times 15) - (5 \times 12)}{(11 \times 15) - (5 \times 5)} = \frac{30}{140} = 0.2143 \text{ (amps)}$$

$$I_2 = \frac{\det \begin{bmatrix} 11 & 6 \\ 5 & 12 \end{bmatrix}}{\det \begin{bmatrix} 11 & 5 \\ 5 & 15 \end{bmatrix}} = \frac{(11 \times 12) - (6 \times 5)}{(11 \times 15) - (5 \times 5)} = \frac{102}{140} = 0.7286 \text{ (amps)}$$