

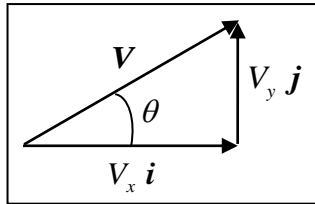
ENGR 1990 Engineering Mathematics

Equation Sheet #2 – Vectors/Complex Numbers

2D Vectors

1. Vector Magnitude and Direction

$$|V| = \sqrt{V_x^2 + V_y^2} \quad \theta = \tan^{-1}(V_y/V_x)$$



2. Vector Sum

$$V_1 + V_2 + \dots + V_n = (V_{1x} + V_{2x} + \dots + V_{nx})i + (V_{1y} + V_{2y} + \dots + V_{ny})j$$

3. Scalar (Dot) Product of Two Vectors

$$A \cdot B = |A||B| \cos(A, B) = (a_x i + a_y j) \cdot (b_x i + b_y j) = a_x b_x + a_y b_y$$

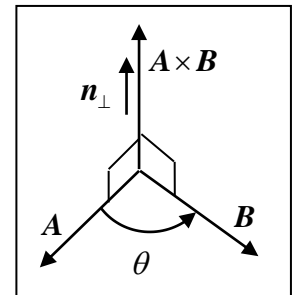
4. Vector Components Parallel and Perpendicular to a Unit Vector n

$$A_{\parallel} = (A \cdot n)n \quad \text{and} \quad A_{\perp} = A - A_{\parallel}$$

5. Vector (Cross) Product of Two Vectors

$$A \times B = (|A||B| \sin(A, B))n_{\perp} = (a_x i + a_y j) \times (b_x i + b_y j) = (a_x b_y - a_y b_x)k$$

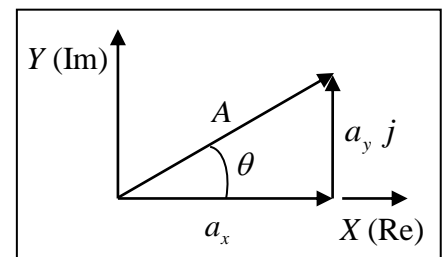
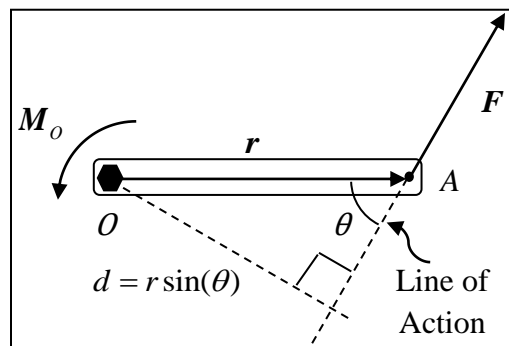
$$A \times B = \begin{vmatrix} i & j & k \\ a_x & a_y & 0 \\ b_x & b_y & 0 \end{vmatrix} = (a_x b_y - a_y b_x)k$$



6. Moment of a Force

$$M_o = r \times F$$

$$d = \frac{|M_o|}{|F|}$$



Complex Numbers ($j = \sqrt{-1}$)

1. Magnitude and Direction

$$A = a_x + ja_y = |A|e^{j\theta} = |A|\angle\theta \quad |A| = \sqrt{a_x^2 + a_y^2} \quad \theta = \tan^{-1}(a_y/a_x) \quad e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

2. Sum

$$A_1 + A_2 + \dots + A_n = (a_{1x} + a_{2x} + \dots + a_{nx}) + j(a_{1y} + a_{2y} + \dots + a_{ny})$$

3. Products and Ratios

$$A \cdot B = (|A|e^{j\alpha})(|B|e^{j\beta}) = |A||B|e^{j(\alpha+\beta)} \quad A/B = (|A|e^{j\alpha})/(|B|e^{j\beta}) = (|A|/|B|)e^{j(\alpha-\beta)}$$