ENGR 1990 Engineering Mathematics
Equation Sheet #3 – Systems of Equations/Exponential, Sine, Cosine Functions

Systems of Equations and Matrices

1. Linear Algebraic Equations: $[A]\{x\}=\{b\}$

$$[A]_{n\times n}=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \qquad \{x\}=\begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} \qquad \{b\}=\begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{Bmatrix}$$

2. Cramer's Rule for $n=2$

$$x_1=\frac{\det\begin{bmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{bmatrix}}{\det[A]} \qquad x_2=\frac{\det\begin{bmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{bmatrix}}{\det[A]} \qquad \det[A]=a_{11}a_{22}-a_{12}a_{21}$$

3. Matrix Inversion

$$[A]\{x\}=\{b\} \quad \Rightarrow \quad \{x\}=[A]^{-1}\{b\}$$

$[A]^{-1}$ exists only if $\det[A]\neq 0$

For 2×2 matrices:

$$[A]=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \Rightarrow [A]^{-1}=\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \Big/ \det[A]=\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \Big/ (a_{11}a_{22}-a_{12}a_{21})$$

Exponential, Natural Logarithms, Sine, and Cosine Functions

1. Exponential and Natural Log

$f(x)=e^x$ $\quad (e=2.71828\ldots)$

Natural Log is the Inverse Function: $\log_e(e^x)=\ln(e^x)=x$ $\qquad e^{\log_e(x)}=e^{\ln(x)}=x$

2. Useful Sine and Cosine Identities

$$\sin(\alpha+\beta)=\sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta)$$

$$\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$$

3. Sines and Cosines

$$x(t)=A\sin(\omega t)+B\cos(\omega t)=M\sin(\omega t+\phi) \qquad M=\sqrt{A^2+B^2} \qquad \phi=\tan^{-1}(B/A)$$

$$x(t)=A\sin(\omega t)+B\cos(\omega t)=M\cos(\omega t+\phi) \qquad M=\sqrt{A^2+B^2} \qquad \phi=\tan^{-1}(-A/B)$$

Frequency: ω (rad/s) $\quad f=\frac{\omega}{2\pi}$ (Hz) $\qquad$ Period: $T=\frac{1}{f}$ (s/cycle)