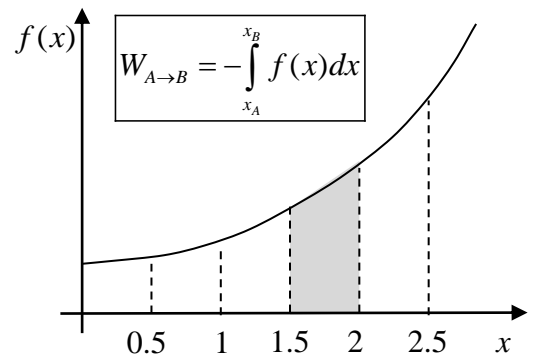


ENGR 1990 Engineering Mathematics

Homework #10 – Integrals

1. A hardening spring has the force-displacement function $f(x) = 100 + 10x + x^2$ (lb). The work done by the spring as it is stretched over some displacement interval is the negative of the integral of the force-displacement function over that interval. Estimate the integral and the work done by the spring as it is stretched from $x = 0$ to $x = 2.5$ inches by breaking the area into a sequence of trapezoids of width $\Delta x = 0.5$ (in).

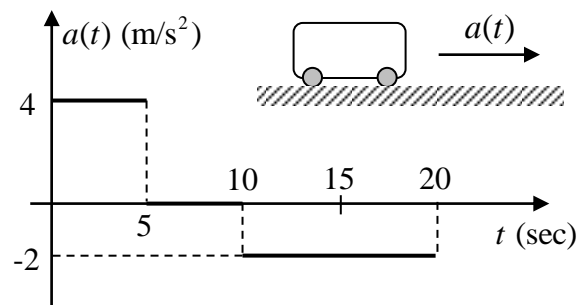
x	$f(x)$	Interval	f_{avg}	Δx
0		--		
0.5		1		0.5
1		2		0.5
1.5		3		0.5
2		4		0.5
2.5		5		0.5
		Σ		



2. Using the same spring force-displacement function given in problem (1), find the work done by the spring using anti-derivatives. Calculate the percent error of the estimate found in problem (1).
3. A car has the acceleration profile shown, and its initial position and velocity are zero. Given that

$$v(t) = \int a(t) dt \quad \text{and} \quad s(t) = \int v(t) dt,$$

find (a) the velocity function $v(t)$, (b) the displacement function $s(t)$, and (c) the total distance traveled by the car for $0 \leq t \leq 20$ (sec). Sketch the functions.



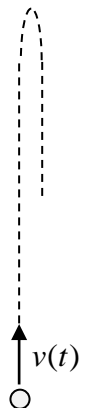
4. A ball that is thrown upward has velocity $v(t) = 75 - 32.2t$ (ft/s). Given that the displacement function of the ball is

$$s(t) = \int v(t) dt,$$

find (a) the displacement of the ball from its original position after 3.5 seconds, and (b) the total distance traveled by the ball during the 3.5 second period.

5. A ball is thrown upward with an initial velocity of $v_0 = 20$ (m/s) from an initial height of $y_0 = 8$ (m) and has a constant downward acceleration of $a_0 = -9.81$ (m/s²).

Given that $v(t) = \int a(t) dt$ and $y(t) = \int v(t) dt$, find (a) the velocity function $v(t)$, and (b) the position function $y(t)$.

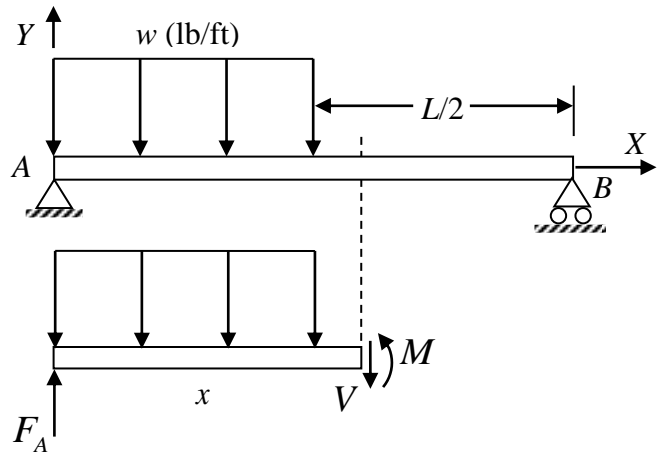


6. The simply supported beam has a uniformly distributed load over the left half of the beam. For a beam of length $L=10$ (ft) and a load $w=100$ (lb/ft), the internal shearing force is

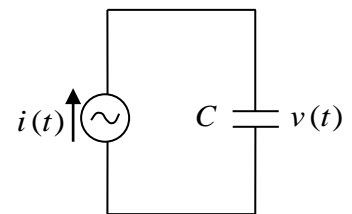
$$\boxed{V(x) = 375 - 100x \text{ (lb)}} \quad 0 \leq x \leq 5 \text{ (ft)}$$

$$\boxed{V(x) = -125 \text{ (lb)}} \quad 5 \leq x \leq 10 \text{ (ft)}$$

Given that the internal bending moments at A and B are zero, find $M(x) = \int V(x)dx$ the internal bending moment as a function of x .



7. A current $i(t) = 2e^{-t}$ (amps) is applied to a capacitor with capacitance $C = 0.25$ (f). Given that $v(t) = \frac{1}{C} \int i(t) dt$, find the
- voltage $v(t)$. Assume $v(0) = 0$.
 - power, $p(t) = v(t) \cdot i(t)$.
 - total energy, $W(t) = \int_0^t p(t) dt$ (joules).



What is the energy at $t = 1, 2, 3, 4$ (sec)? What is the limit of the energy as $t \rightarrow \infty$?

8. A voltage $v(t) = 10 \sin(120\pi t)$ (volts) is applied to an inductor with inductance $L = 250$ (mh). Find the current $i(t)$, given that $i(t) = \frac{1}{L} \int v(t) dt$. Assume $i(0) = 0$.

