

## ENGR 1990 Engineering Mathematics

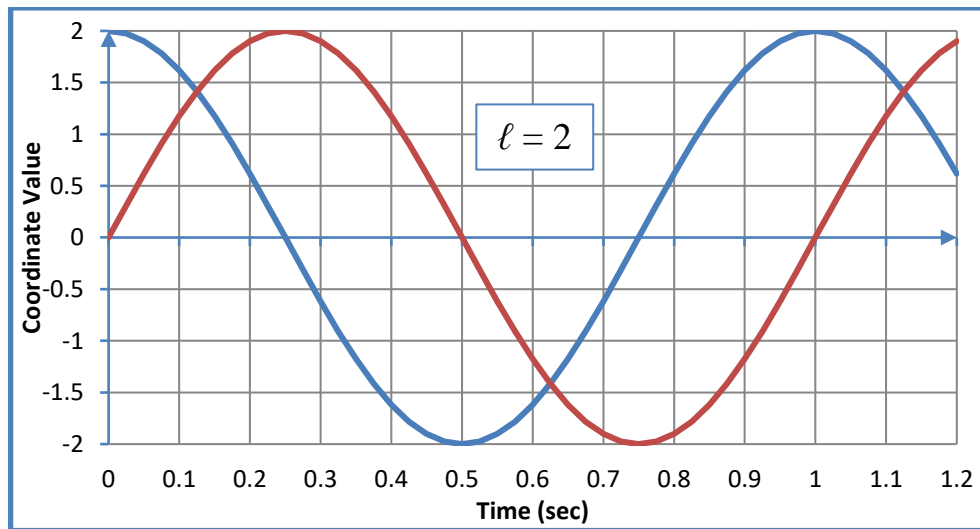
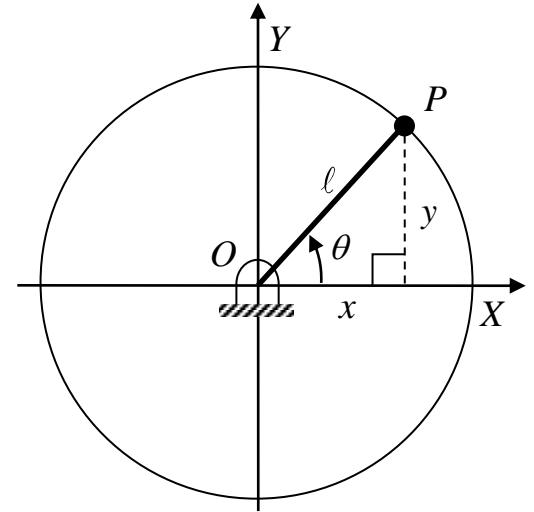
### Sine and Cosine Functions of Time

Arm  $OP$  rotates so  $P$  moves in a circular path. From trigonometry, the coordinates of  $P$  are

$$x = \ell \cos(\theta) \quad y = \ell \sin(\theta)$$

If the bar completes one revolution ( $2\pi$  radians) in one second, then  $\theta = 2\pi t$ . So, we can also express the coordinates as functions of time.

$$x = \ell \cos(2\pi t) \quad y = \ell \sin(2\pi t)$$



Characteristic	Symbol
Amplitude	$A = \ell$
Frequency	$\omega = 2\pi$ (radians/sec)
	$f = \omega/2\pi$ (cycles/sec) -or- (Hertz (Hz))
Period	$T = 1/f$ (seconds/cycle)

Note that the sine and cosine functions (and hence the  $X$  and  $Y$  coordinates) may be related by a **phase shift**. The phase shift corresponds to a **shift in time**.

$$\sin(2\pi t) = \cos\left(2\pi t - \pi/2\right) \quad \cos(2\pi t) = \sin\left(2\pi t + \pi/2\right)$$

In this case, the phase shift of  $\pi/2$  radians equates to a shift in time of  $1/4$  second. For any given phase shift, the corresponding time shift depends on the frequency  $\omega$ .