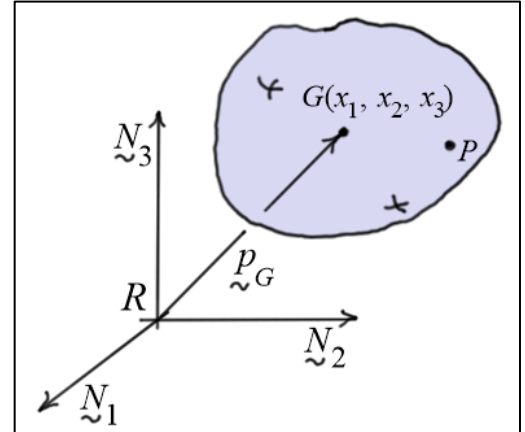


ME 6590 Multibody Dynamics Accelerations Using Absolute Coordinates

Mass Center Accelerations

Consider the *rigid body* in the diagram. Given that (x_1, x_2, x_3) are the position coordinates of the mass center G relative to the fixed frame, then \underline{p}_G the position vector of G , ${}^R \underline{v}_G$ the velocity of G , and ${}^R \underline{a}_G$ the acceleration of G in R may be written in matrix form as



$$\{p_G\} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} \quad \{v_G\} = \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} \quad \{a_G\} = \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix}$$

Accelerations of Other Points

We can write the inertial components of the velocities of other points in the system, such as P in the diagram above, in terms of the inertial components of \underline{v}_G the velocity of G as follows:

$$\{v_P\} = \{v_G\} + [\tilde{\omega}_B][R]^T \{p'_{P/G}\}$$

$$\{v_P\} = \{v_G\} + [R]^T [\tilde{\omega}'_B] \{p'_{P/G}\}$$

The first equation is written in terms of the fixed-frame components of ${}^R \underline{\omega}_B$ and the second is written in terms of the body-fixed components of ${}^R \underline{\omega}_B$. These two equations can be *differentiated* to give the fixed-frame components of ${}^R \underline{a}_P$ the acceleration of P .

$$\{a_P\} = \{a_G\} + [\dot{\tilde{\omega}}_B][R]^T \{p'_{P/G}\} + [\tilde{\omega}_B][\dot{R}]^T \{p'_{P/G}\}$$

$$= \{a_G\} + [\tilde{\alpha}_B][R]^T \{p'_{P/G}\} + [\tilde{\omega}_B][\tilde{\omega}_B][R]^T \{p'_{P/G}\}$$
(1)

$$\{a_P\} = \{a_G\} + [R]^T [\dot{\tilde{\omega}}'_B] \{p'_{P/G}\} + [\dot{R}]^T [\tilde{\omega}'_B] \{p'_{P/G}\}$$

$$= \{a_G\} + [R]^T [\tilde{\alpha}'_B] \{p'_{P/G}\} + [R]^T [\tilde{\omega}'_B][\tilde{\omega}'_B] \{p'_{P/G}\}$$
(2)