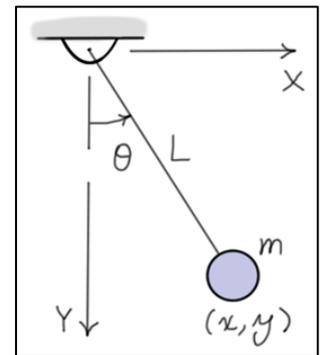


## ME 6590 Multibody Dynamics

### Four Simulink Models of a Simple Pendulum

In these notes, *four Simulink models* of a simple pendulum are presented. The models are intended to show the solution process *with* and *without configuration constraints*. The first model uses a *single* ordinary differential equation for the angle  $\theta$ ; the second uses a *pair* of ordinary differential equations for the coordinates  $x$  and  $y$  *decoupled* in the second derivative terms; the third uses a *pair* of *coupled* ordinary differential equations for the coordinates  $x$  and  $y$ ; and the fourth uses *three coupled* ordinary differential equations for the coordinates  $x$  and  $y$  and a Lagrange multiplier  $\lambda$ .

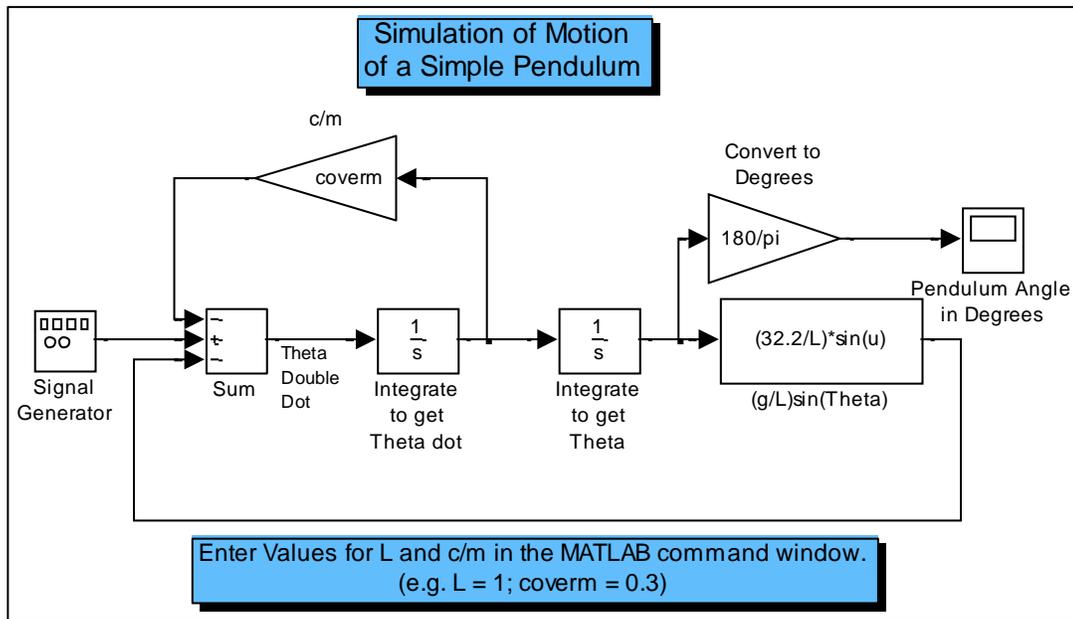
See the notes entitled “**Lagrange’s Equations for MDOF Systems with Constraints**” for the development of the equations used in these models. In three of the four models shown here, a *damping term* was added to slow the motion of the pendulum.



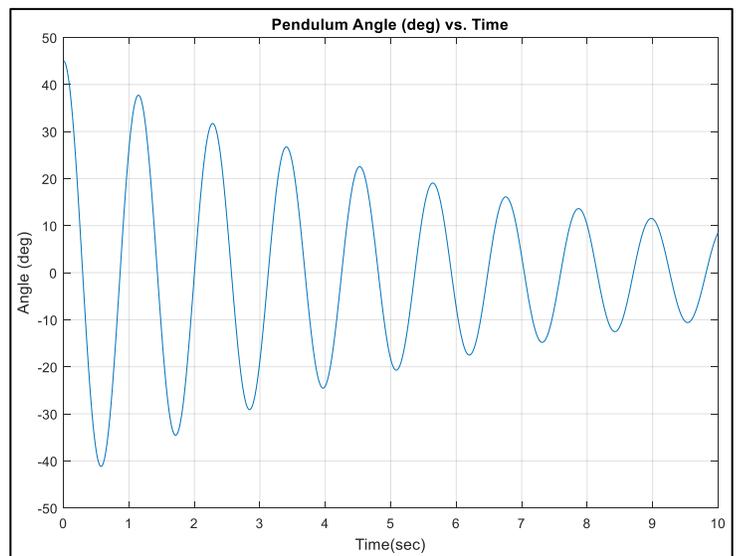
Note that for the *simple pendulum*, the obvious choice of models is Model 1. It is, in fact, the simplest. However, for *more complex systems*, it may not always be convenient (or possible) to *eliminate* the *extra generalized coordinates*. In these cases, it may be more convenient to develop the equations of motion in terms of a set of *Lagrange multipliers*. It may also not be convenient to *eliminate* the Lagrange multipliers or to *decouple* the differential equations in the second derivatives. In such a case, the approach of Model 4 may be the best choice. The four different approaches are applied here to a simple problem to make it easier to *compare* them.

**Model 1:** Using a single differential equation for the angle  $\theta$ .

The Simulink diagram of Model 1 is shown below. The *two closed loops* represent the *differential equation*. The two feedback loops are associated with *damping* and *gravity*. A signal generator was added for *external excitation* of the pendulum, and the angle is converted to degrees for plotting.

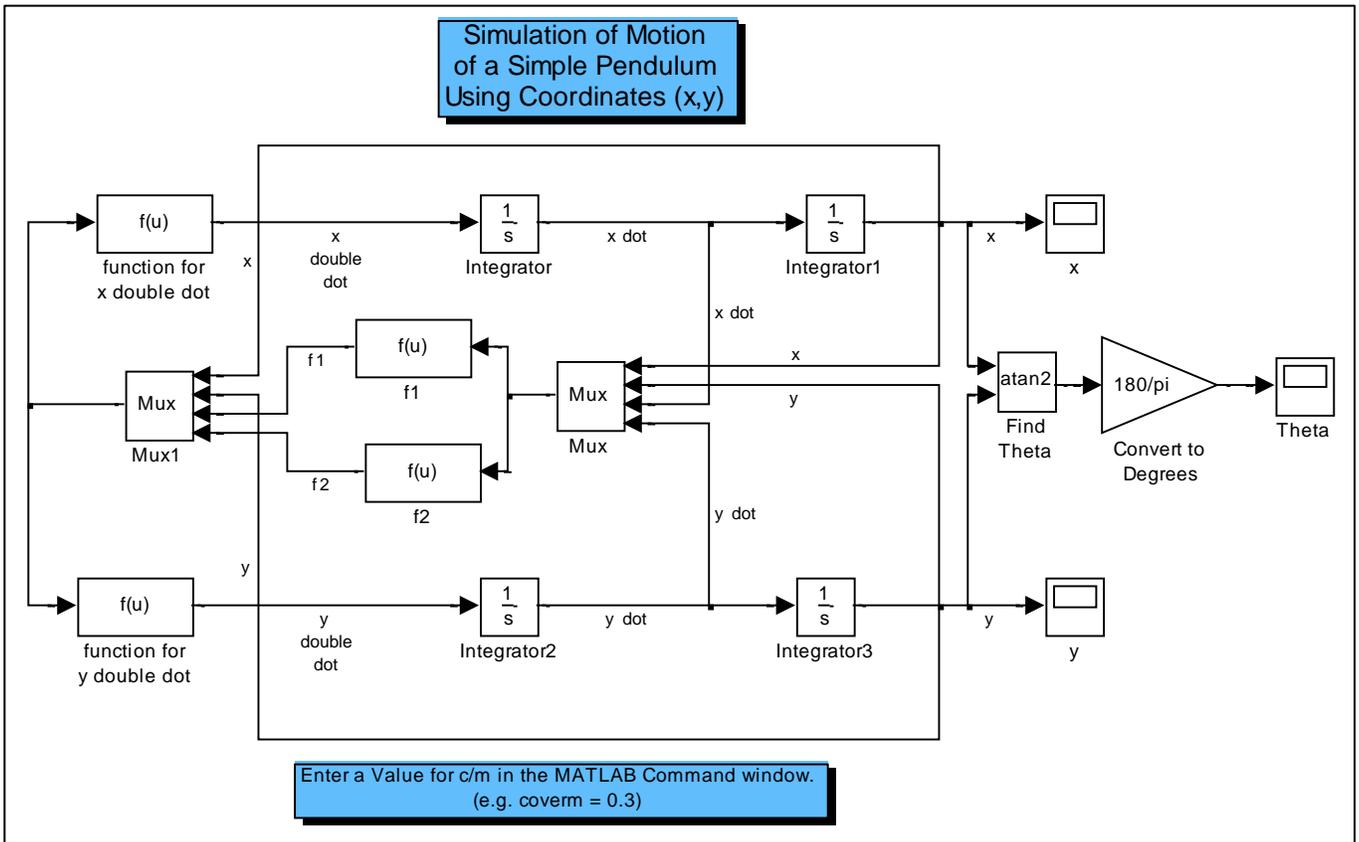


The results shown to the right are for a pendulum of length  $L = 1$  (ft), a rotational damping factor to mass ratio  $\frac{c}{m} = 0.3$ , an initial angle of release of  $\theta_i = 45$  (deg), and no external excitation. The scope data (the calculated angle  $\theta(t)$ ) was exported to the MATLAB workspace for plotting.

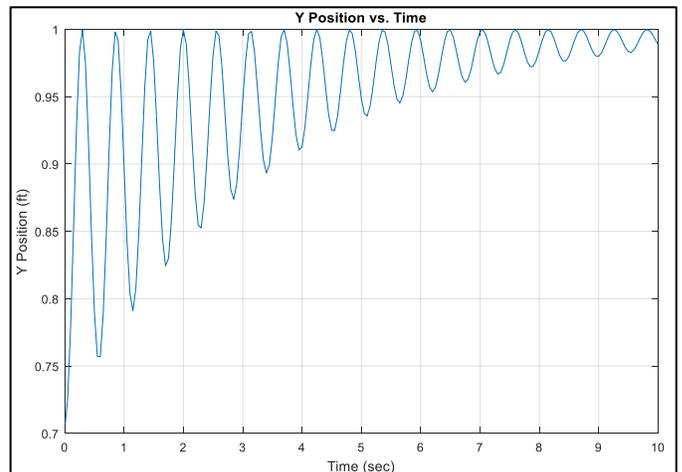
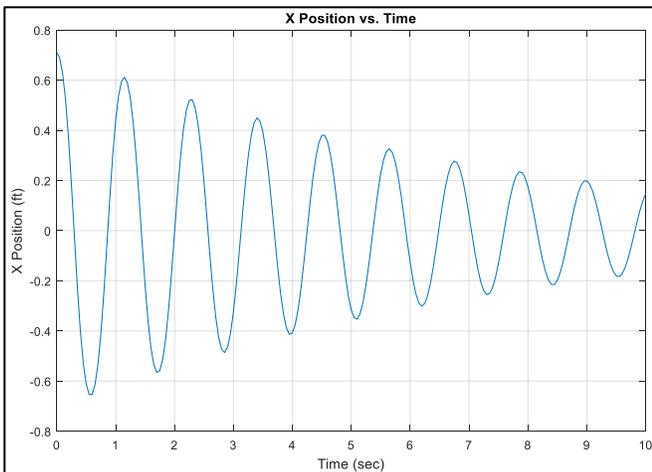


**Model 2:** Using two *decoupled* equations for the coordinates  $(x, y)$

The Simulink diagram for Model 2 is shown below. In this model, two differential equations decoupled in their second derivatives are solved for the coordinates  $x$  and  $y$ . The output of the model includes the pendulum angle  $\theta$ . These two equations have been decoupled in the second derivative terms, so  $\ddot{x}$  and  $\ddot{y}$  can be calculated as independent functions of  $(x, \dot{x}, y, \dot{y})$ .

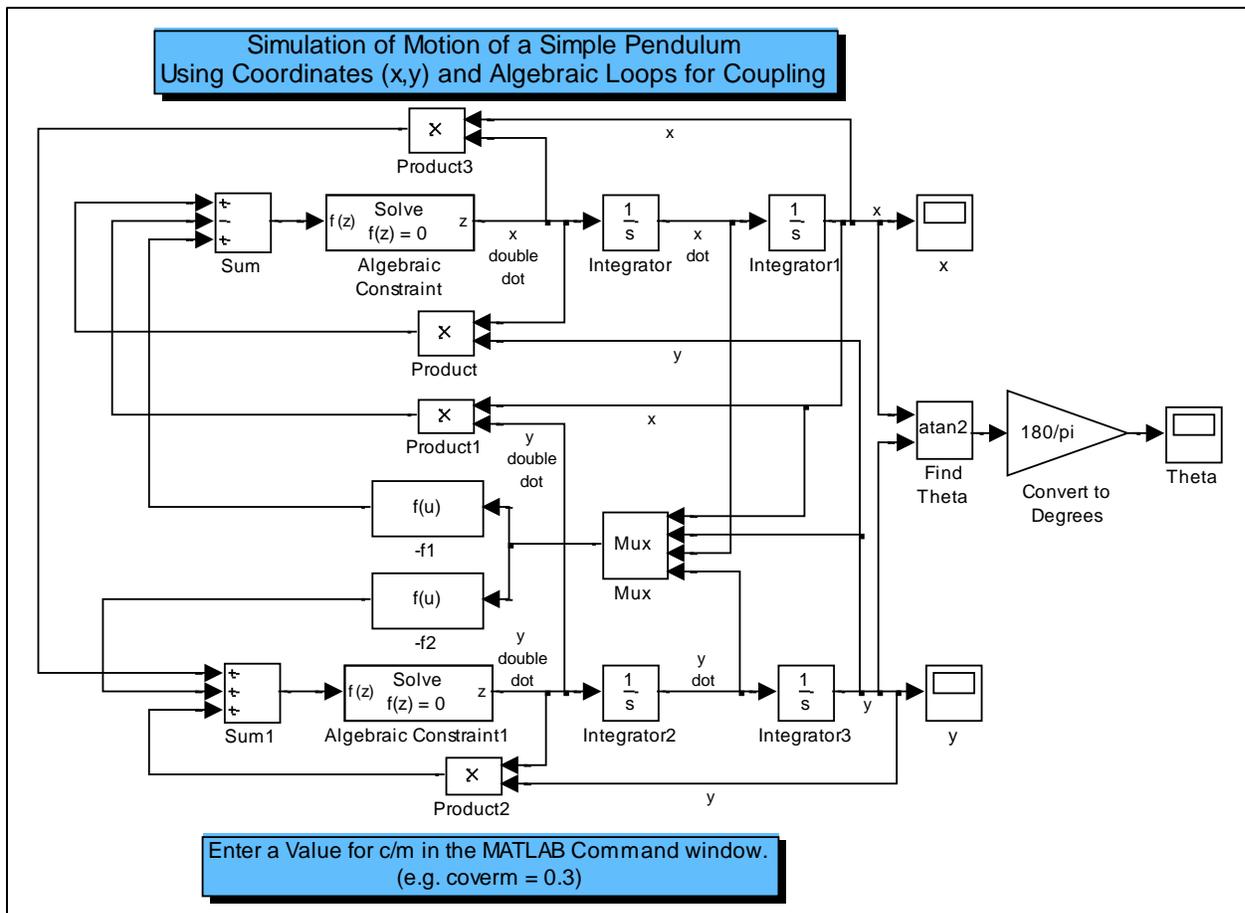


Using the *same* parameters and initial condition as for Model 1, the following results were produced for the coordinates  $x$  and  $y$ . As expected, the results for the angle  $\theta$  are identical to those of Model 1.



**Model 3:** Using two coupled equations for the coordinates  $(x, y)$ .

The Simulink diagram for Model 3 is shown below. In this model, the two differential equations for the coordinates  $x$  and  $y$  have *not been decoupled* in the second derivatives. In this case, a set of two *linear algebraic equations* must be solved at each time step to find the second derivatives  $\ddot{x}$  and  $\ddot{y}$ . This process is accomplished in Simulink using “Algebraic Constraint” loops. The loops iterate on the values of  $\ddot{x}$  and  $\ddot{y}$  until the algebraic equations are solved. This saves the analyst the work of having to decouple the equations which can be quite tedious as the number of equations increases. The results for this model are identical to those of Model 2.



**Model 4:** Using three coupled equations for coordinates and Lagrange multiplier  $(x, y, \lambda)$

The Simulink diagrams for Model 4 are shown in the five panels below. In this model, the differential equations are *coupled* in the *second derivatives* and include an algebraic unknown *Lagrange multiplier*  $\lambda$ . The first panel on the left shows the *top layer*

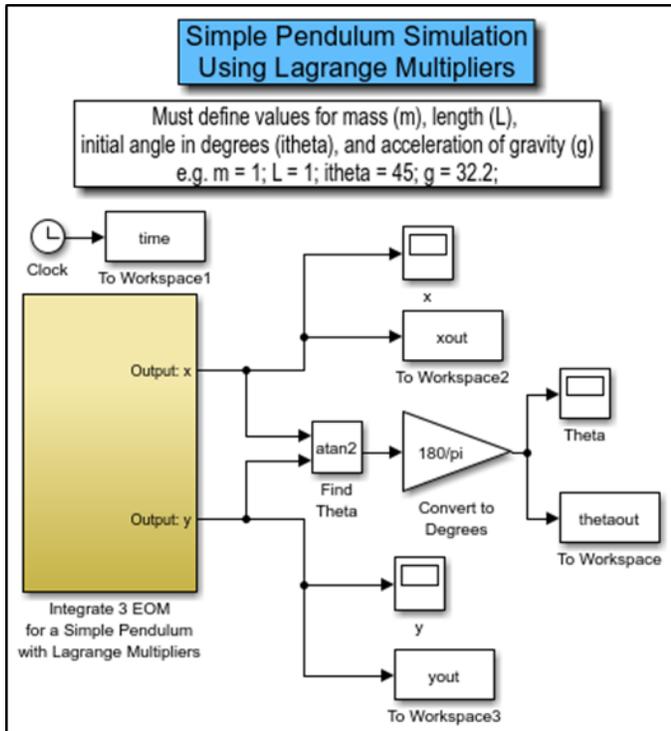
of the model. The equations are solved in the *tan-colored subsystem* on the left. The output of this subsystem are the coordinates  $x$  and  $y$ . Those coordinates are used to calculate the angle  $\theta$ . All three variables are sent to the MATLAB workspace for plotting. Note that the *Lagrange multiplier* is also calculated in this subsystem, but it is *not* included in the output of the subsystem.

The details of the solution process are shown in the next four panels. The panel with the *green subsystems* simply shows the coupling between the three equations – the  $x$  equation, the  $y$  equation, and the constraint equation. The last three panels show the contents of each of the three green equation subsystems. Note that the loop associated with each of the three differential/algebraic equations has an “Algebraic Constraint” block because the equations are coupled in the second derivatives and the Lagrange multiplier.

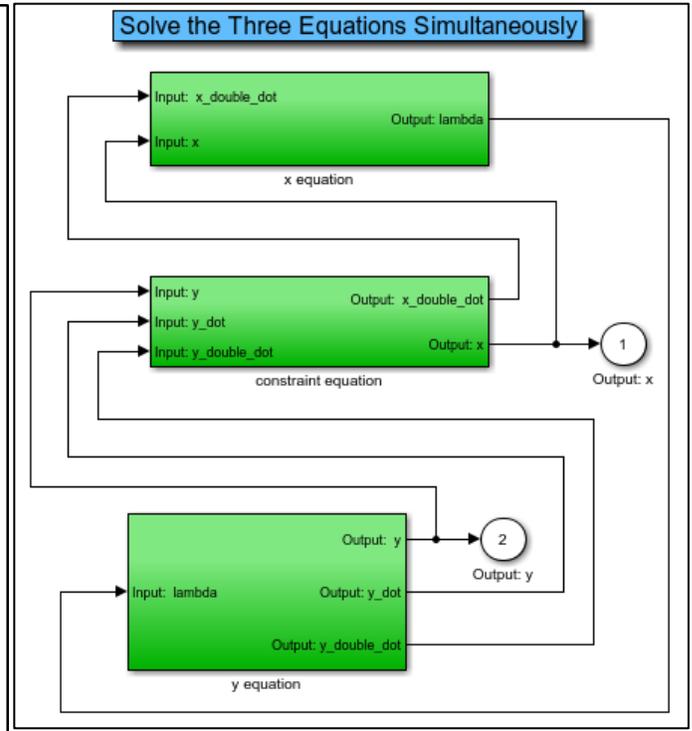
As suggested in Panel 1 below, the mass was set to 1 (slug), the length to 1 (ft), and the initial angle to 45 (deg), and the acceleration of gravity was set to  $32.2 \text{ (ft/s}^2\text{)}$ . The effect of *damping* was *not included*.

The equations of motion developed for this model are *more complicated* than the other three models; however, this is *not always the case* for more complicated systems. Using this approach for more complex systems can produce a simpler set of equations.

Because there is no damping included in this model, the pendulum angle oscillates between  $\pm 45$  (deg). Also, because this model includes the Lagrange multiplier, the tension in the pendulum connector is also calculated. (Recall that the Lagrange multiplier is related to the *tension per unit length* in the pendulum connector (i.e.  $\lambda = -T/L$ .) The length of the pendulum is set to 1 (ft), so the value of the Lagrange multiplier is equal to the negative of the tension in the connector in pounds. The mass of the pendulum is  $m = 1$  (slug), so it weighs 32.2 (lb.). Note from the graph that the Lagrange multiplier (negative of the tension) varies from about  $-23$  (lb.) to  $-51$  (lb.). In the initial position, the tension should be equal to  $W \cos(45) \approx 22.8$  (lb.).



Panel 1: Top layer of the model



Panel 2: Contents of the tan-colored subsystem

