

ME 2580 Example #44: (Rigid Body Kinetics – General Plane Motion Example #2)

Given: $\ell = 2$ (ft), $W_{AB} = 4$ (lb), $W_B = 10$ (lb)

system is released from rest at $\theta = 45$ (deg)

Find: a_B , the acceleration of block B at the instant of release

Solution:

Writing Newton's laws of motion using the free-body diagrams gives

Block B :

$$\rightarrow \sum F_x = B_x = \left(\frac{W_B}{g}\right) a_{B_x} \quad (1)$$

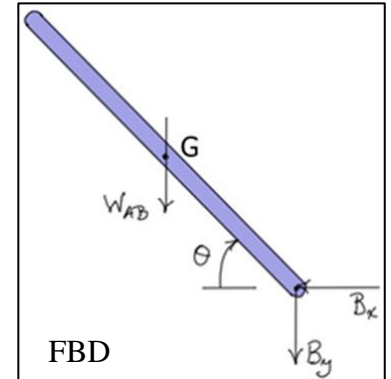
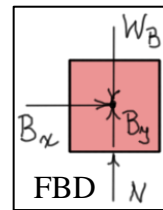
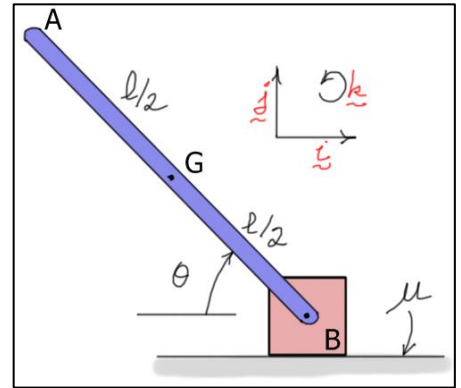
$$+\uparrow \sum F_y = B_y + N - W_B = 0 \quad (2)$$

Bar AB :

$$\rightarrow \sum F_x = -B_x = \left(\frac{W_{AB}}{g}\right) a_{G_x} \quad (3)$$

$$+\uparrow \sum F_y = -B_y - W_{AB} = \left(\frac{W_{AB}}{g}\right) a_{G_y} \quad (4)$$

$$\curvearrowright \sum M_G = -\left(\frac{\ell}{2} \sin(45)\right) B_x - \left(\frac{\ell}{2} \cos(45)\right) B_y = I_G \alpha = \frac{1}{12} \left(\frac{W_{AB}}{g}\right) \ell^2 \alpha \quad (5)$$



These are five equations in seven unknowns ($B_x, B_y, N, a_{G_x}, a_{G_y}, \alpha, a_B$). Two more equations are required. A kinematic analysis provides these equations.

Kinematics:

The angular acceleration α , the acceleration a_B , and the components of acceleration a_G can be related using the concept of relative acceleration.

$$\begin{aligned} \underline{a}_G &= \underline{a}_B + \underline{a}_{G/B} = a_B \underline{i} + (\alpha \underline{k} \times \underline{r}_{G/B}) - \cancel{\omega_{AB}^2} \underline{r}_{G/B} = (a_B \underline{i}) + \alpha \underline{k} \times \left(-\frac{\ell}{2} \cos(45) \underline{i} + \frac{\ell}{2} \sin(45) \underline{j}\right) \\ &= \left(a_B - \frac{\ell}{2} \alpha \sin(45)\right) \underline{i} + \left(-\frac{\ell}{2} \alpha \cos(45)\right) \underline{j} \end{aligned}$$

$$\Rightarrow \boxed{a_{G_x} = a_B - \frac{\ell}{2} \alpha \sin(45)} \quad \boxed{a_{G_y} = -\frac{\ell}{2} \alpha \cos(45)}$$

Using this last result, a_{G_x} and a_{G_y} can be eliminated from Eqs. (1), (3), (4), and (5) to get four equations in four unknowns.

$$\begin{aligned}
 B_x - \left(\frac{W_B}{g}\right)a_B &= 0 \\
 -B_x - \left(\frac{W_{AB}}{g}\right)\left(a_B - \frac{\ell}{2}\alpha \sin(45)\right) &= 0 \\
 -B_y - \left(\frac{W_{AB}}{g}\right)\left(-\frac{\ell}{2}\alpha \cos(45)\right) &= W_{AB} \\
 -\left(\frac{\ell}{2}\sin(45)\right)B_x - \left(\frac{\ell}{2}\cos(45)\right)B_y - \frac{1}{12}\left(\frac{W_{AB}}{g}\right)\ell^2\alpha &= 0
 \end{aligned}$$

Solving gives

$$\begin{aligned}
 B_x &= 1.2 \text{ (lb)} \\
 B_y &= -2.32 \text{ (lb)} \\
 a_B &= 3.86 \text{ (ft/s}^2\text{)} \rightarrow \\
 \alpha &= 19.1 \text{ (r/s}^2\text{)} \curvearrowright
 \end{aligned}$$