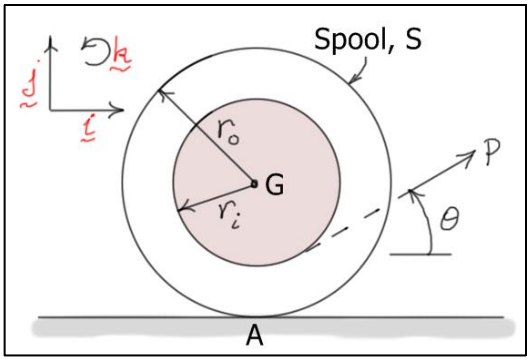
ME 2580 Example #45b: (Rigid Body Kinetics – Work and Energy Example #2)



Given: $r_o = 0.4$ (m), $r_i = 0.25$ (m), $m = 100$ (kg)

$P = 200$ (N), $\theta = 20$ (deg), $k_G = 0.3$ (m)

Spool rolls without slipping; released from rest

Find: ω the angular velocity of S after G moves 2 (m) to the right.

Solution: (A is the instantaneous center (IC) of the spool)

$$\boxed{KE_1 + U_{1\to 2} = KE_2} \quad \text{(initial state: 1, final state: 2)}$$

with

$$\boxed{KE_1 = 0} \quad \text{(released from rest)}$$

$$\boxed{KE_2 = \tfrac{1}{2} I_A \omega_2^2 = \tfrac{1}{2}\left(I_G + m r_o^2\right)\omega_2^2 = \tfrac{1}{2}\left(m k_G^2 + m r_o^2\right)\omega_2^2 = 12.5\,\omega_2^2}$$

The spool ***rolls without slipping***, so the only force that does work is P. The ***distance*** d_P over which P acts can be calculated as follows.

$$\boxed{d_P = \underbrace{\Delta x_G \cos(\theta)}_{\text{translation of S}} - \underbrace{r_i\,\Delta\theta}_{\text{rotation of S}} = 2\cos(\theta) - r_i\left(\frac{\Delta x_G}{r_o}\right) = 2\cos(20^\circ) - 0.25\left(\frac{2}{0.4}\right) \approx 0.629385 \text{ (m)}}$$

Using this result, the ***work done*** by P can be calculated as follows.

$$\boxed{U_{1\to 2} = P\,d_P \approx 200(0.629385) \approx 125.877 \text{ (N-m)}}$$

Substituting into the work-energy equation gives: $\boxed{\omega_2 \approx \sqrt{\frac{125.877}{12.5}} \approx 3.17335 \approx 3.17 \text{ (rad/s)}}$.

As the spool rolls to the ***right***, the angular motion is ***clock-wise***.

Note:

From an ***earlier analysis*** using ***Newton's law***, the ***angular acceleration*** of the spool was calculated to be $\boxed{\alpha \approx 1.00702 \text{ (rad/s}^2\text{)}}$. Given a ***constant angular acceleration***, the change in angular velocity can be calculated as follows.

$$\boxed{\omega_2^2 = \omega_1^2 + 2\alpha\,\Delta\theta = 2\alpha\left(\frac{\Delta x_G}{r_o}\right) \approx 2(1.00702)\left(\frac{2}{0.4}\right) \approx 3.17336 \approx 3.17 \text{ (rad/s)}}$$

Question:

It is ***assumed*** here that the spool ***rolls without slipping***. How can this assumption be ***verified***? Answer: Newton's law. See Example #43.