ME 2580 Example #46: (Rigid Body Kinetics – Conservation of Energy Example)

Given: *AB* and *BC* are slender rods, $\ell = 3$ (ft)
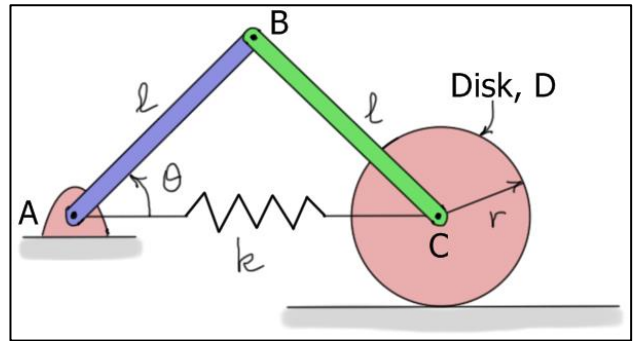
$W_{AB} = W_{BC} = W = 15$ (lb), $W_D = 20$ (lb)

$k = 4$ (lb/ft)

spring is unstretched at $\theta = 45$ (deg)

released from rest at $\theta = 45$ (deg)

Disk rolls without slipping, $r = 1$ (ft)

Find: ω_{AB} the angular velocity of *AB* at $\theta = 0$ (deg)

Solution:

Using the **conservation of energy** on the system (state 1: $\theta = 45$ (deg); state 2: $\theta = 0$ (deg))

$$\boxed{\underset{\text{zero}}{K_1} + V_1 = K_2 + V_2}$$ (released from rest)

Defining a datum at the horizontal line through *A* and *C* gives

$$\boxed{V_1 = \left(V_1\right)_{\text{gravity}} + \underset{\text{zero}}{\underbrace{\left(V_1\right)_{\text{spring}}}} = 2\left(W\tfrac{\ell}{2}\sin(45)\right) \approx 31.8198 \text{ (ft-lb)}}$$

$$\boxed{V_2 = \underset{\text{zero}}{\underbrace{\left(V_2\right)_{\text{gravity}}}} + \left(V_2\right)_{\text{spring}} = \tfrac{1}{2}k\left(2\ell - 2\ell\cos(45)\right)^2 = 2k\ell^2\left(1 - \cos(45)\right)^2 \approx 6.17662 \text{ (ft-lb)}}$$

In state 2, the disk has **zero velocity**, *AB* is rotating about *A*, and the **instantaneous center** of *BC* is at *C*. Also, due to the **symmetry** of the mechanism, the angular velocities of *AB* and *BC* are **equal** (but in opposite directions).

$$\boxed{\begin{aligned} K_2 &= \left(K_2\right)_{\text{disk}} + \left(K_2\right)_{AB} + \left(K_2\right)_{BC} = \underset{(v_C)_2 = 0}{\underbrace{0}} + \tfrac{1}{2}I_A\omega_{AB}^2 + \tfrac{1}{2}I_C\omega_{BC}^2 \\ &= \tfrac{1}{2}\left(\tfrac{1}{3}\left(\tfrac{W}{g}\right)\ell^2\right)\omega_{AB}^2 + \tfrac{1}{2}\left(\tfrac{1}{3}\left(\tfrac{W}{g}\right)\ell^2\right)\omega_{AB}^2 = \tfrac{1}{3}\left(\tfrac{W}{g}\right)\ell^2\omega_{AB}^2 \approx 1.39752\omega_{AB}^2 \end{aligned}}$$

Substituting into the conservation of energy equation gives

$$31.8198 = 6.17662 + 1.39752\,\omega_{AB}^2 \quad \Rightarrow \quad \boxed{\omega_{AB} = \sqrt{\frac{31.8198 - 6.17662}{1.39752}} \approx 4.28 \text{ (rad/s)}}$$

The motion of *AB* is **clockwise**, and the motion of *BC* is **counter-clockwise**.

Question: What other forces (besides the spring and gravity) affect the motion of this system? Why is the work done by these forces on the system equal to zero?