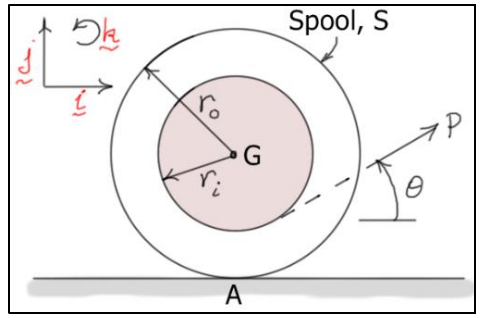
ME 2580 Example #47: (Rigid Body Kinetics – Impulse & Momentum Example #1)

Given: $r_o = 0.4$ (m), $r_i = 0.25$ (m), $m = 100$ (kg), $k_G = 0.3$ (m)

$P = 200$ (N) , $\theta = 20$ (deg)

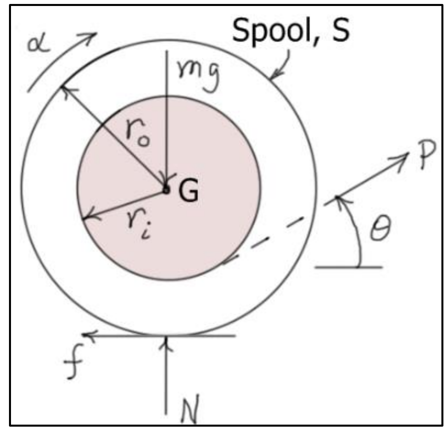
released from ***rest*** with $\mu_s = 0.2$, $\mu_k = 0.15$

Find: ω, the angular velocity of spool *S* after 3 seconds



Solution:

The spool is ***released from rest*** and when the force *P* is applied all ***reaction forces*** are ***constant***. Applying the principles of linear and angular impulse and momentum to the free body diagram gives



$$\underbrace{\underset{\sim}{L}_1}_{\text{zero}} + \sum\int \underset{\sim}{F}\,dt = \underset{\sim}{L}_2 \qquad \underbrace{(\underset{\sim}{H}_G)_1}_{\text{zero}} + \sum\int \underset{\sim}{M}_G\,dt = (\underset{\sim}{H}_G)_2$$

$$\xrightarrow{+} \sum\int F_x dt = \left(P\cos(\theta) - f\right)\Delta t = m\left(v_G\right)_{2x} \qquad (1)$$

$$+\uparrow \sum\int F_y dt = \left(P\sin(\theta) + N - mg\right)\Delta t = m\left(v_G\right)_{2y} = 0 \qquad (2)$$

$$\circlearrowright\!+ \sum\int M_G dt = \left(r_o\, f - r_i\, P\right)\Delta t = I_G\omega_2 = m k_G^2\, \omega_2 \qquad (3)$$

Assuming the spool ***rolls without slipping***, $(v_G)_{2x}$ and ω_2 are related as follows.

$$(v_G)_{2x} = r_o\,\omega_2$$

After ***substituting*** for $(v_G)_{2x}$ in Eq. (1) and ***rearranging*** terms, Eqs. (1) and (3) can be written

$$\begin{aligned} (m\, r_o)\omega_2 \quad + (\Delta t)\, f &= P\cos(\theta)\Delta t \approx 563.8156 \\ (m\, k_G^2)\omega_2 - (r_o\,\Delta t)\, f &= -r_i\, P\,\Delta t = -150 \end{aligned}$$

Solving gives: $\omega_2 \approx 3.02$ (rad/s) (clockwise) $f \approx 147.658 \approx 148$ (lb)

Check: Using Eq. (2), $f_{max} = \mu_s N = \mu_s\left(mg - P\sin(\theta)\right) \approx 183 \text{ (N)} > f$ $\Rightarrow$ no slipping occurs

Note:

To ***apply*** the principles of ***linear*** and ***angular impulse and momentum*** to a problem, the ***impulses*** of the forces and moments must be calculated. If the forces and moments are ***position*** (and hence, ***time***) ***dependent***, it may be ***quite difficult*** to calculate the linear and angular impulses. In these cases, it may ***not*** be ***practical*** to use these principles.