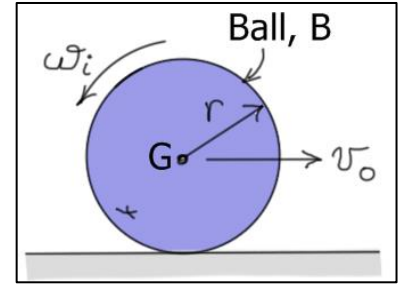


ME 2580 Example #49: (Rigid Body Kinetics – Impulse & Momentum Example #3)

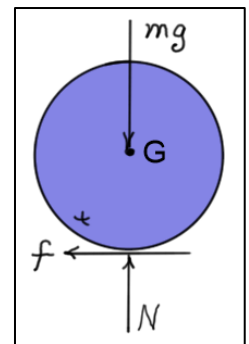
Given: $m_{\text{ball}} = m = 5$ (kg); $r = 0.1$ (m); $\mu_k = 0.08$
 initial backspin, $\omega_i = 10$ (rad/s) (counter-clockwise)
 initial velocity, $v_o = 5$ (m/s) (to the right)



Find: a) Δt_b , the time it takes for ball to stop back spinning and the corresponding velocity of G
 b) Δt_r , the time it takes for ball to start rolling without slipping and the corresponding velocity of G

Solution:

The summation of vertical forces gives $N = mg$. Because the ball is slipping during the entire motion, $f = \mu_k N = \mu_k mg$.



a) Applying the principles of linear and angular momentum gives

$$\vec{+} (L_1)_x + \sum (\text{Imp})_x = (L_2)_x \quad (1: \text{initial state, } 2: \text{backspin stops})$$

$$(mv_G)_1 - f \Delta t_b = (mv_G)_2$$

$$\vec{+} (H_G)_1 + \sum \int M_G dt = (H_G)_2 \quad I_G = \frac{2}{5} mr^2 = \frac{2}{5} \times 5 \times 0.1^2 = 0.02 \text{ (kg}\cdot\text{m}^2)$$

$$I_G \omega_1 + r f \Delta t_b = I_G \omega_2 = 0 \quad (\text{angular velocity is } \mathbf{zero} \text{ at instant back spinning stops})$$

$$0.02(-10) + 0.1 \mu_k m g \Delta t_b = 0 \Rightarrow \Delta t_b = \frac{0.2}{(0.1)(0.08)(5)(9.81)} \approx 0.50968 \approx 0.510 \text{ (sec)}$$

Substituting for Δt_b in the equation for linear momentum gives

$$(v_G)_2 = \frac{1}{m}(mv_o - f \Delta t_b) = v_o - \mu_k g \Delta t_b = 5 - (0.08 \times 9.81 \times \Delta t_b) = 4.6 \text{ (m/s)}$$

b) (1: initial state, 3: ball begins to roll without slipping) $\Rightarrow (v_G)_3 = r \omega_3$

$$(mv_G)_1 - f \Delta t_r = (mv_G)_3 \Rightarrow 5(v_G)_3 + (0.08 \times 5 \times 9.81) \Delta t_r = mv_o = 25$$

$$I_G \omega_1 + r f \Delta t_r = I_G \omega_3 \Rightarrow \left(\frac{0.02}{0.1}\right)(v_G)_3 - (0.1 \times 0.08 \times 5 \times 9.81) \Delta t_r = 0.02 \times (-10) = -0.2$$

Solving the last two equations simultaneously gives

$$(v_G)_3 \approx 3.28571 \approx 3.29 \text{ (m/s)}$$

$$\Delta t_r \approx 2.18436 \approx 2.18 \text{ (sec)}$$