

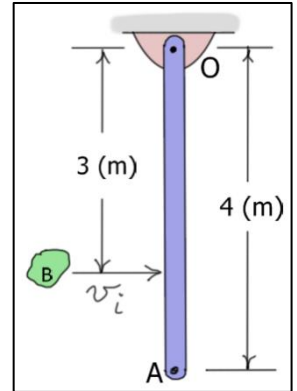
ME 2580 Example #50: (Rigid Body Kinetics – Impulse & Momentum – Impact Example #1)

Given: bar  $OA$  is initially at rest when the blob  $B$  strikes it  
 $B$  sticks to the bar and they move together after impact  
 $m_{OA} = 15$  (kg);  $m_B = 5$  (kg);  $v_i = 10$  (m/s)

Find:  $\theta_{\max}$ , the maximum angle the bar reaches after impact

Solution:

During the impact,  $\sum M_O = 0$ , so the **angular momentum** of the system about  $O$  is **conserved**.

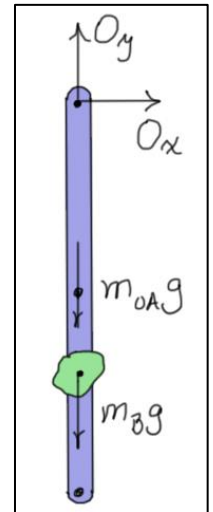


$$\overset{\curvearrowright}{(H_O)_1} = (H_O)_2 \quad (\text{state 1: just before impact; state 2: just after impact})$$

where

$$(H_O)_1 = 3(m_B v_i) = 3 \times 5 \times 10 = 150 \text{ (N-m-s)}$$

$$(H_O)_2 = 3(m_B v_B)_2 + I_O \omega_2 = 3m_B (3\omega_2) + \left(\frac{1}{3} m_{OA} \ell^2\right) \omega_2 = \left(45 + \left(\frac{1}{3} \times 15 \times 4^2\right)\right) \omega_2 = 125 \omega_2$$



Substituting into the conservation of angular momentum equation gives

$$\omega_2 = 150 / 125 = 1.2 \text{ (rad/s)} \quad (\text{angular velocity of } OA \text{ just after impact})$$

To find the **maximum angle** the bar reaches, apply the **conservation of energy**. Defining the **datum** of the **bar** at the bottom-most position of  $G$ , and the **datum** of  $B$  at its bottom-most position and observing the **kinetic energy** of the system is **zero** at the **maximum angle**, write

$$\underbrace{KE_2}_{\text{zero}} + \underbrace{V_2}_{\text{zero}} = \underbrace{KE_3}_{\text{zero}} + \underbrace{V_3}_{\text{zero}} \quad (\text{state 2: just after impact; state 3: at maximum angle})$$

where

$$KE_2 = \frac{1}{2} I_O \omega_2^2 + \frac{1}{2} m_B (v_B^2)_2 = \frac{1}{2} \left(\frac{1}{3} m_{OA} \ell^2\right) \omega_2^2 + \frac{1}{2} m_B (3\omega_2)^2 = \frac{1}{2} (80) (1.2)^2 + \frac{1}{2} (5) (1.2)^2 = 90 \text{ (N-m)}$$

$$V_3 = m_{OA} g (2 - 2 \cos(\theta_{\max})) + m_B g (3 - 3 \cos(\theta_{\max})) = (2m_{OA} + 3m_B) g (1 - \cos(\theta_{\max})) = 45g (1 - \cos(\theta_{\max}))$$

Substituting and solving:

$$45g (1 - \cos(\theta_{\max})) = 90 \Rightarrow 1 - \cos(\theta_{\max}) = 90 / (45g) \Rightarrow \theta_{\max} = \cos^{-1} \left( \frac{45g - 90}{45g} \right)$$

$$\Rightarrow \theta_{\max} \approx 37.24 \text{ (deg)}$$