ME 3600 Control Systems
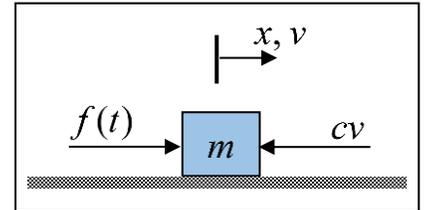**Proportional Control of a First Order System**

Example System: Simple Speed Control System

Consider a **car** of **mass** m traveling along a road with **wind resistance** (proportional to the speed of the car) as shown in the diagram. Applying Newton's 2nd law in the direction of travel and neglecting friction, write

$$\overset{+}{\rightarrow} \sum F = f(t) - cv = m\dot{v} \qquad \Rightarrow \boxed{m\dot{v} + cv = f(t)}$$
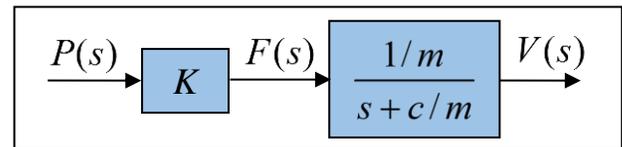

Using **Laplace transforms**, the **transfer function** for the system is

$$\boxed{\frac{V}{F}(s) = \frac{1/m}{s + c/m}} \qquad (1)$$

Open-Loop, Proportional Speed Control

Now consider **proportional, open-loop speed control** of the car as indicated in the block diagram. The system **transfer function** is

$$\boxed{\frac{V}{P}(s) = \frac{K/m}{s + c/m}} \qquad (2)$$

The **final value** due to a **unit step** input $P(s) = 1/s$ is $\boxed{v_{ss} = (K/m)/(c/m) = K/c}$.

Fig. 1 shows the step response of this system for $K = 300$, 600, and 900 using the parameters shown in Eq. (3) below. Note the value of K **affects** the **magnitude** of the response, but it **does not affect** how long the car takes to reach a new final speed. That is, it **does not affect** the system's **settling time**.
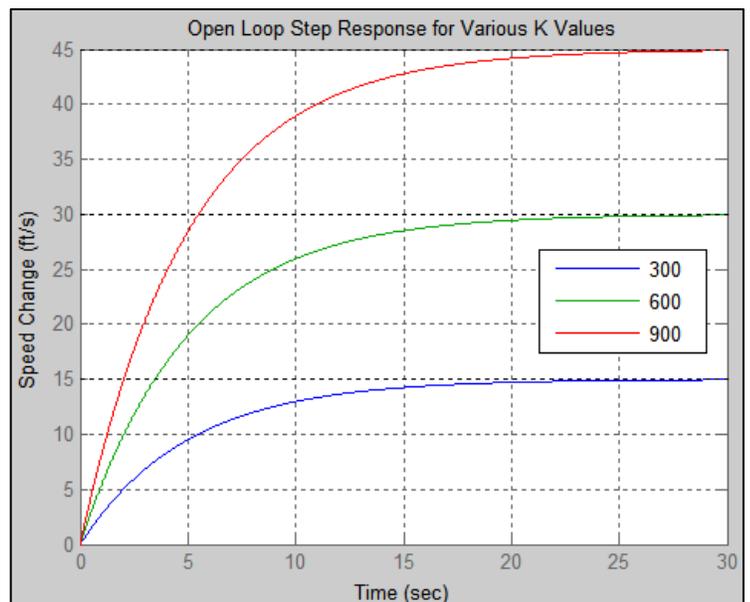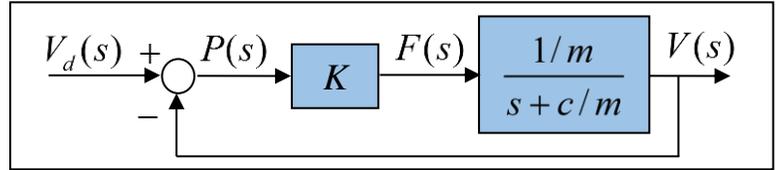

Fig. 1 Open-Loop Step Response for Various K Values

$$\boxed{\begin{array}{l} m = 100 \text{ slugs} \\ c = 20 \text{ (lb-s/ft)} \end{array}} \qquad (3)$$

Closed-Loop, Proportional Speed Control

Finally, consider the **proportional, closed-loop speed control** of the car as indicated in the block diagram. The transfer function of this system is

$$\frac{V}{V_d}(s) = \frac{K/m}{s+c/m+K/m} = \frac{K/m}{s+(c+K)/m} \tag{4}$$

The system input is V_d the desired speed, and the output is the actual speed. The final value due to a *step input* $V_d(s) = 1/s$ is $\boxed{v_{ss} = (K/m)/\left((c+K)/m\right) = K/(c+K)}$. Fig. 2 shows the step response of the system for $K = 300$, 600, and 900. Note that the value of K **affects both** the **magnitude** of the response and the **time** it takes the car to reach a new final speed. **Higher** values of K give **shorter** settling times.
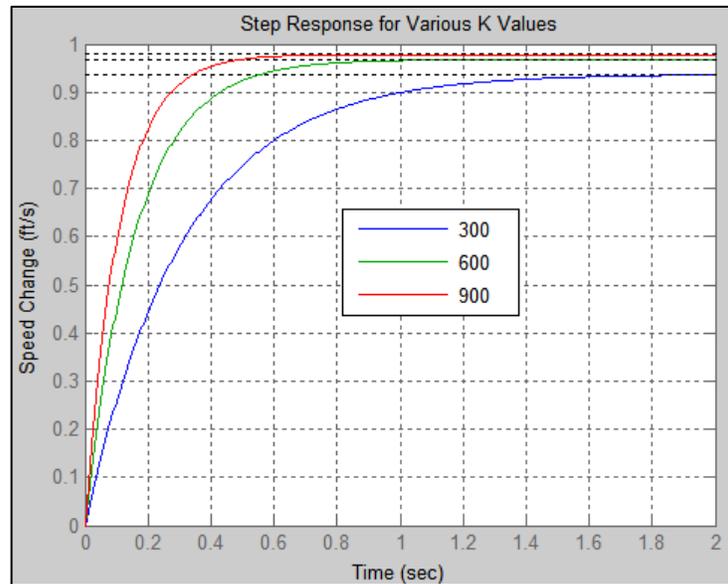

Fig. 2 Closed-Loop Step Response for Various K

In theory, the value of K could be **increased** further to make the steady state response (v_{ss}) closer to the commanded value $(=1 \text{ (ft/s)})$ and the settling time smaller and smaller. However, as these changes are made, the **force** required to move the car becomes higher and higher. Fig. 3 shows the driving force $f(t)$ associated with the **unit step responses** shown in Fig. 2. Clearly, higher velocity commands and higher gains will cause the **forces** to eventually become **unrealistic**.
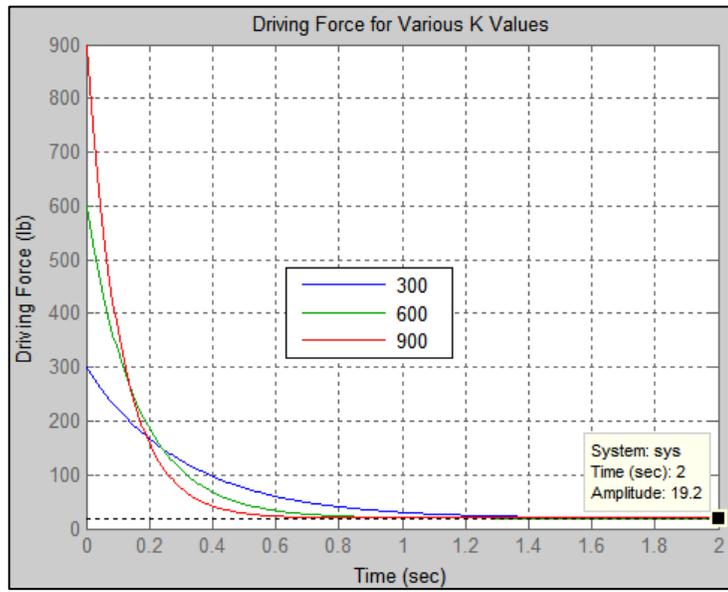
Fig. 3 Driving Force for Closed-Loop Step Response for Various Gains