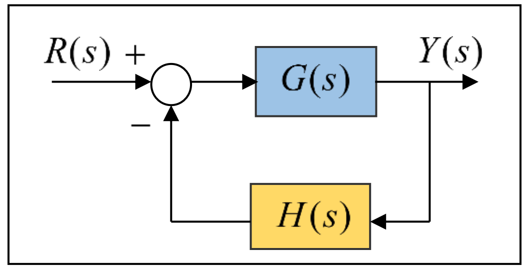
ME 3600 Control Systems
Root Locus Diagrams

1. Write the characteristic equation of the closed-loop system, $\boxed{1 + GH(s) = 0}$. Then algebraically rewrite the equation in the form $\boxed{1 + kP(s) = 0}$. Here, k is the root locus parameter. $P(s)$ should be in the form of a ratio with polynomials of s in the numerator and denominator.



Simple Closed Loop System

2. Note that the numerator of $P(s)$ must have a ***plus sign*** on the highest power of s.

   Root Locus (RL): $0 \le k \le +\infty$

   Complementary Root Locus (CRL): $-\infty \le k \le 0$

3. Find the poles and zeros of $P(s)$. The root loci (paths) ***start*** at the ***poles*** and ***proceed*** to the ***zeros*** as k advances from 0 to $\pm\infty$. The ***number of branches*** of the root loci is ***equal to*** the number of poles. That is, a separate branch starts at each pole of $P(s)$. (n_p is the number of poles, and n_z is the number of zeros.)

4. ***Every point*** on the ***branches*** of the ***root locus*** must satisfy the ***angle condition***:

   The difference between the sum of the angles of the vectors drawn to the point from the poles of $P(s)$ and the sum of the angles of the vectors drawn to it from the zeros of $P(s)$ is an odd multiple of 180 degrees.

5. ***Every point*** on the ***branches*** of the ***complementary root locus*** must satisfy the ***angle condition***:

   The difference between the sum of the angles of the vectors drawn to the point from the poles of $P(s)$ and the sum of the angles of the vectors drawn to it from the zeros of $P(s)$ is an even multiple of 180 degrees, including 0 degrees.

6. The root loci are symmetric with respect to the real axis.

7. ***All parts*** of the ***real axis*** are either on the root locus or on the complementary root locus diagrams.

   Root Locus: The segments of the real axis that have an ***odd*** number of poles and zeros of $P(s)$ to the right are on branches of the ***root locus diagram***.

   Complementary Root Locus: The segments of the real axis that have an ***even*** (or zero) number of poles and zeros of $P(s)$ to the right are on the branches of the ***complementary root locus diagram***.

8. Angles of Asymptotes of the Root Locus Branches.

   Root Locus: For ***large values*** of k the branches that proceed to ***infinity*** do so along ***asymptotes*** given by the ***angles***:

$$\boxed{\varphi_A = \left[\frac{2m+1}{n_p - n_z}\right] 180^\circ} \qquad \left(m = 0,\ 1,\ 2,\ \ldots\ ,\ n_p - n_z - 1\right)$$

   Complementary Root Locus: For ***large values*** of k the branches that proceed to ***infinity*** do so along ***asymptotes*** given by the ***angles***:

$$\boxed{\varphi_A = \left[\frac{2m}{n_p - n_z}\right] 180^\circ} \qquad \left(m = 0,\ 1,\ 2,\ \ldots\ ,\ n_p - n_z - 1\right)$$

9. ***Intersection*** of the ***Asymptotes*** with the ***real axis***: The ***asymptotes*** of the branches of the root locus diagram ***all intersect*** at a ***single point*** on the ***real axis***. The following formula gives the location of this intersection point.

$$\boxed{\sigma_A = \frac{\Sigma(\text{pole locations}) - \Sigma(\text{zero locations})}{n_p - n_z}}$$

   Note that for complex conjugate poles and zeros, the imaginary parts cancel from the summation, so only the real parts of the locations of the poles and zeros must be included.

10. Break-away (or Break-in) Points (if any):

Define: $\boxed{p(s) = -1/P(s)}$ Set: $\boxed{\dfrac{dp(s)}{ds} = 0}$ and solve for s.

All real solutions are either break-away (or break-in) points on the root locus or on the complementary root locus diagrams. ***Break-away points*** are those locations on the real axis where the poles ***leave the axis*** as k advances, and ***break-in points*** are those points where the poles ***move onto*** the axis as k advances.

11. Angles of Departure and Arrival of the Root Loci:

The angle of the tangent line at any point along the branch of a root locus or complementary root locus diagram must satisfy the angle conditions listed in (4) and (5) above. At a pole of $P(s)$, the angle is called an ***angle of departure***. At a zero of $P(s)$, the angle is called an ***angle of arrival***.