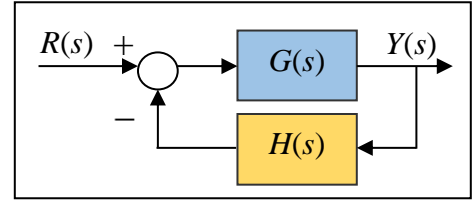


ME 3600 Control Systems
Summary Procedure for the Root Locus Diagram



Simple Closed Loop System

1. Write the characteristic equation: $1 + GH(s) = 0$

Rewrite the equation in the form: $1 + kP(s) = 0$

Root Locus (RL): $0 \leq k \leq +\infty$

2. Find the **poles** and **zeros** of $P(s)$. The root loci start at the poles and proceed to the zeros as k advances from $0 \rightarrow \infty$.

- The number of branches (loci) on the RL diagram is equal to n_p , the number of poles.
- The number of asymptotes is $n_A = n_p - n_z$. (n_z is the number of zeros)

3. Plot the **pole-zero diagram** for $P(s)$. Then,

- Identify those segments of the real axis that contain roots of the characteristic equation. There are roots on those segments such that an **odd** number of poles and zeros are to the right of that segment.
- Identify the direction of movement of the poles. The poles of the closed loop system move from the poles of $P(s)$ to the zeros of $P(s)$ (or to infinity) as K increases from $0 \rightarrow \infty$.

4. Calculate the **angles of all asymptotes** (if any):

$$\phi_A = \left[\frac{2m+1}{n_p - n_z} \right] 180^\circ \quad (m = 0, 1, 2, \dots, (n_p - n_z - 1))$$

5. Calculate the **intersection point** of the asymptotes with the real axis (if any):

$$\sigma_A = \frac{\sum(\text{pole locations}) - \sum(\text{zero locations})}{n_p - n_z}$$

6. **Sketch** the branches of the root locus diagram. Keep in mind that the root loci are **symmetric** with respect to the **real axis**.

7. Calculate the locations of the **break points** (if any):

Define: $p(s) = -1/P(s)$ Set: $\frac{dp(s)}{ds} = 0$ **or** $\frac{dP(s)}{ds} = 0$ and solve for s .

8. **Angles of Departure** and **Arrival** of the Root Loci:

- The angle of the tangent to the root locus at any point must satisfy the angle condition: The **difference** between the sum of the angles of the vectors drawn to the point from the poles of $P(s)$ and the sum of the angles of the vectors drawn to it from the zeros of $P(s)$ is an **odd multiple of 180°** .
- At a pole of $P(s)$, the angle is called an angle of departure. At a zero of $P(s)$, the angle is called an angle of arrival.