ME 3600 Control Systems
## What is a Root Locus Diagram?

<u>Definition</u>:   A *root locus diagram* is the plot of the *paths* of the poles of a closed loop

system as a single parameter K is varied.

Root Locus Diagram: $0 \le K < +\infty$
<u>Complementary Root Locus Diagram</u>: $-\infty < K \le 0$

<u>Example</u>:

o *Proportional position control* of a *space platform* can be expressed by the block diagram below. The open-loop platform dynamics is *second-order* and *under-damped*.
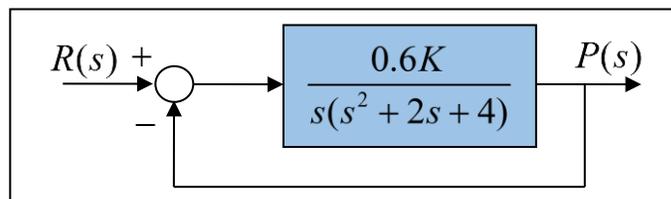
<u>Question</u>: How do the *poles* of this closed-loop system *vary* as the parameter K changes?

o One way to answer this question is simply to find the closed loop transfer function, identify the *characteristic equation*, and compute the *roots* for a variety of parameter values. For this system, the characteristic equation is $\boxed{s^3 + 2s^2 + 4s + 0.6K = 0}$.

o The figure below shows a *plot* of the *poles* of the system for a given set of K values. The red stars indicate the locations of the poles for $1 \le K \le 27$ with an increment $\Delta K = 2$. The blue circles indicate the locations of the poles for $-1 \le K \le -27$ with an increment of $\Delta K = -2$.

Observations about this root locus diagram:

o As K *increases* (becoming more *positive*), the *real pole* moves from the *origin* along the *negative real axis*, while a pair of *complex conjugate poles* crosses over the imaginary axis and move into the *right-half plane*. As K *decreases* (becoming more negative), the *real pole* moves from the origin along the *positive real axis*, while a pair of *complex conjugate poles* move further into the *left-half plane*.

o As K increases toward $+\infty$ or decreases toward $-\infty$ the branches follow *asymptotes*.

o The poles **do not** necessarily **move uniformly** along the branches.

o Although the poles do not move uniformly along the branches, the branches are **smooth curves**.

o Recall that if all the poles of a system are in the **left-half plane**, it is **stable**, and if any of the poles are in the **right-half plane**, it is **unstable**. This system is stable for $0 \le K \le 13$ (approximately). A **more precise estimate** of the upper limit of K can be found using the Routh-Hurwitz method or by specifying smaller increments in MATLAB.