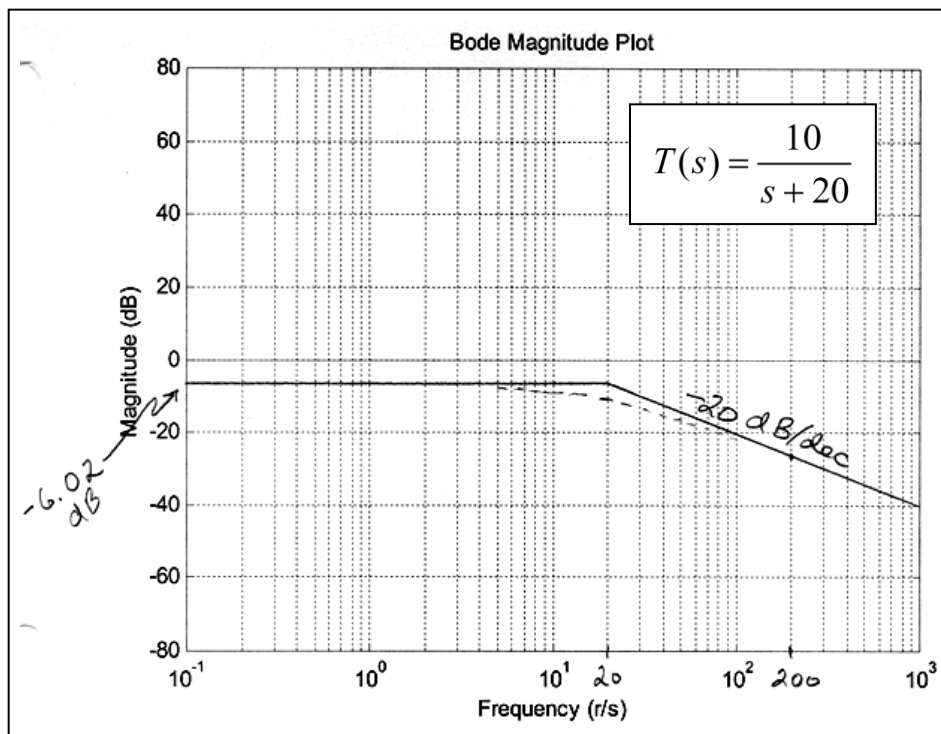


# ME 3600 Control Systems

## Bode Diagram Examples

Example 1:

The system is *first-order* with a gain  $K = 10$  and a real pole at  $s = -20$ . Because the system has *no poles* or *zeros* at the *origin*, the *low-frequency asymptote* is *constant*. The system has a *corner frequency* at 20 (rad/s) at which point the slope *decreases* by 20 (dB/decade). The *dotted line* indicates the system is *less responsive* near the corner frequency than is indicated by the asymptotes. The system has *zero* phase at low frequencies and transitions to  $-90$  (deg) as the corner frequency is passed.



Corner Frequency:  $\omega = 20$  (rad/sec)

Low frequency asymptote:  $\omega \ll 20$  (rad/sec)

$$M(\omega) = \text{constant} = 20\log(10) - 20\log(20) = -6.02 \text{ (dB)}$$

$$\phi = 0 \text{ (deg)}$$

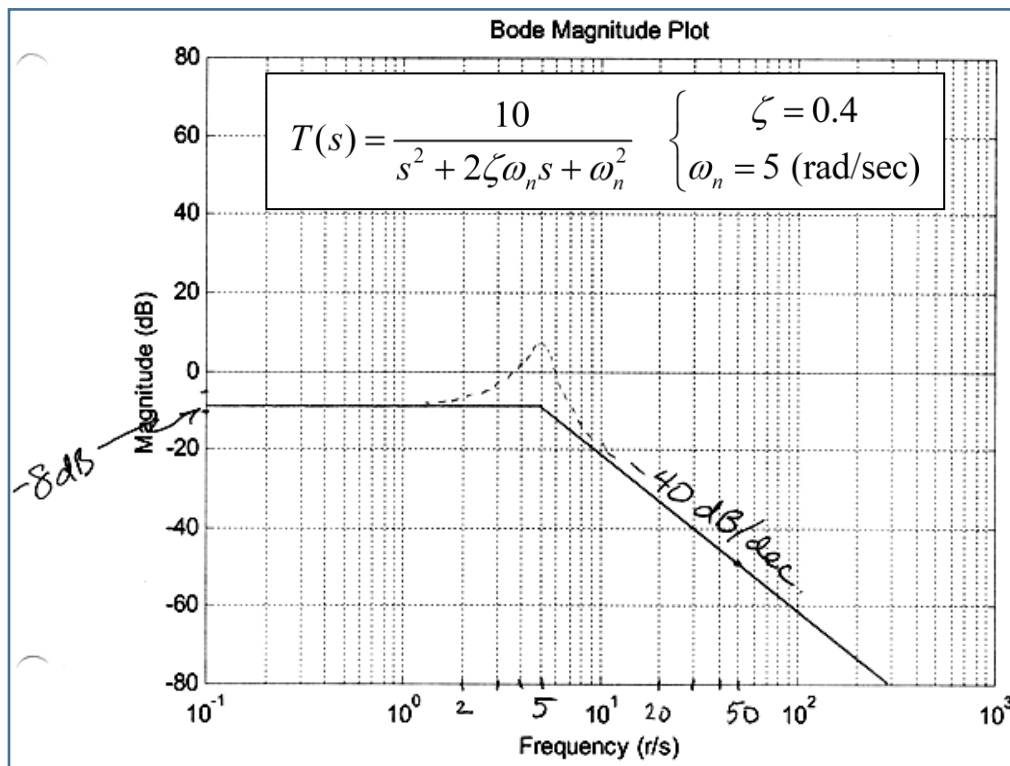
High frequency asymptote:  $\omega \gg 20$  (rad/sec)

$$M(\omega) = -20\log(\omega) \dots \text{ (a } -20 \text{ dB/decade slope)}$$

$$\phi = -90 \text{ (deg)}$$

Example 2:

The system is *second-order* with a gain  $K=10$  and a pair of complex poles with natural frequency  $\omega_n = 5$  (rad/s) and damping ratio  $\zeta = 0.4$ . Because the system has *no poles* or *zeros* at the *origin*, the *low-frequency asymptote* is *constant*. The system has a *corner frequency* at  $\omega = 5$  (rad/s) at which point the slope *decreases* by 40 (dB/decade). The *dotted line* indicates the system is *more responsive* near the corner frequency than is indicated by the asymptotes. The system has *zero* phase at low frequencies and transitions to  $-180$  (deg) as the corner frequency is passed.



Corner Frequency:  $\omega = 5$  (rad/sec)

Low frequency asymptote:  $\omega \ll 5$  (rad/sec)

$$M(\omega) = \text{constant} = 20\log(10) - 40\log(5) = -7.96 \text{ (dB)} \approx -8 \text{ (dB)}$$

$$\phi = 0 \text{ (deg)}$$

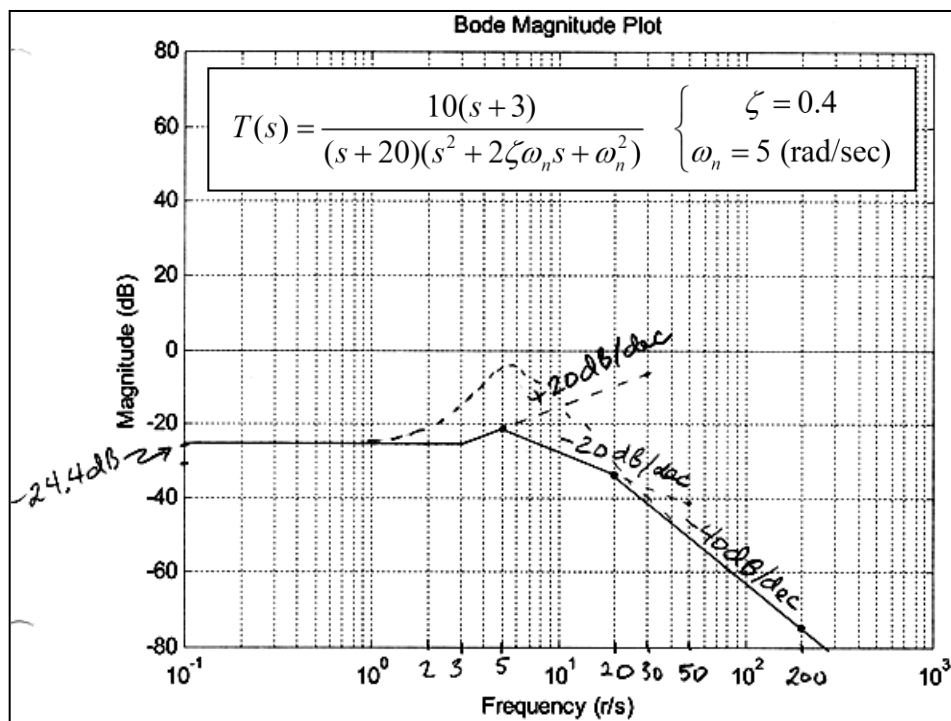
High frequency asymptote:  $\omega \gg 5$  (rad/sec)

$$M(\omega) = -40\log(\omega) \dots \text{ (a } -40 \text{ dB/decade slope)}$$

$$\phi = -180 \text{ (deg)}$$

Example 3:

The system is *third-order* with a gain  $K = 10$ , a real zero at  $s = -3$ , a pair of complex poles with natural frequency  $\omega_n = 5$  (rad/s) and damping ratio  $\zeta = 0.4$ , and a real pole at  $s = -20$ . Because the system has *no poles* or *zeros* at the *origin*, the *low-frequency asymptote* is *constant*. The system has *corner frequencies* at  $\omega = 3, 5, 20$  (rad/s). At 3 (rad/s) the *slope increases* by 20 (dB/decade), at 5 (rad/s) it *decreases* by 40 (dB/decade), and at 20 (rad/s) it *decreases* by another 20 (dB/decade). The *dotted line* indicates the system is *more responsive* near  $\omega_n = 5$  (rad/s) than is indicated by the asymptotes. The response at this frequency is increased by the presence of the zero. The system has *zero* phase at low frequencies and transitions to  $-90$  (deg) and  $-180$  (deg) as the corner frequencies are passed.



Corner Frequencies:  $\omega = 3, 5, 20$  (rad/sec)

Low frequency asymptote:  $\omega \ll 3$  (rad/sec)

$$M(\omega) = \text{constant} = 20\log(10) + 20\log(3) - 20\log(20) - 40\log(5) = -24.4 \text{ (dB)}$$

$$\phi = 0 \text{ (deg)}$$

$3 < \omega < 5$  (rad/sec):  $M(\omega)$  has a +20 dB/decade slope

$5 < \omega < 20$  (rad/sec):  $M(\omega)$  has a -20 dB/decade slope

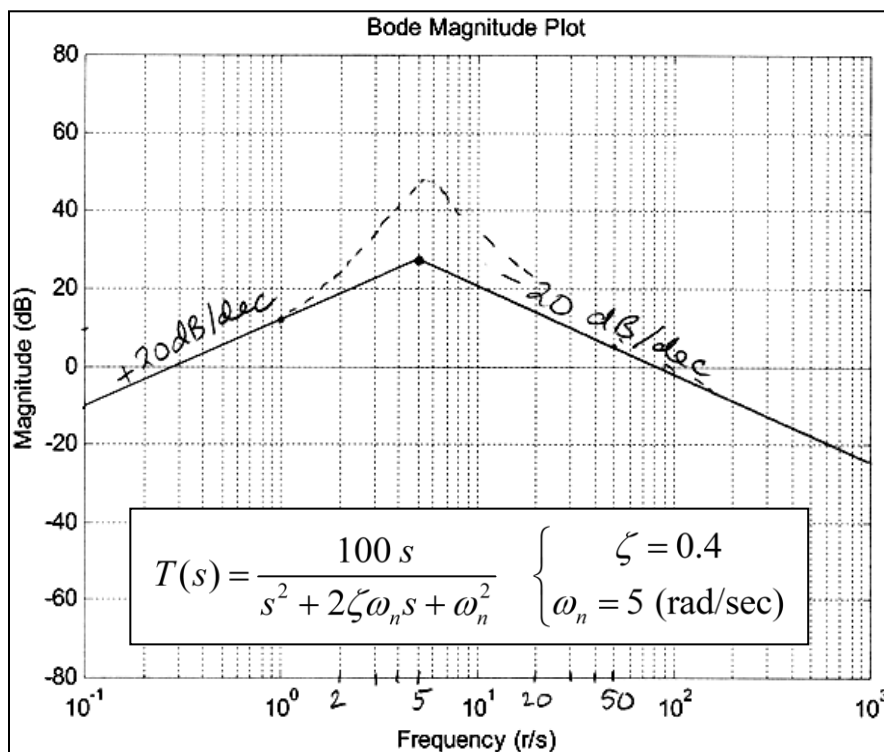
High frequency asymptote:  $\omega \gg 20$  (rad/sec)

$M(\omega)$  has a -40 dB/decade slope

$$\phi = -180 \text{ (deg)}$$

Example 4:

The system is *second-order* with a gain  $K = 100$ , a *zero of order one* at the *origin*, and a *pair of complex poles* with natural frequency  $\omega_n = 5$  (rad/s) and damping ratio  $\zeta = 0.4$ . Because the system has a *zero of order one* at the *origin*, the *low-frequency asymptote* has a slope of  $+20$  (dB/decade). The system has a *corner frequency* at  $\omega = 5$  (rad/s) at which point the slope *decreases* by  $40$  (dB/decade). The *dotted line* indicates the system is *more responsive* near the corner frequency than is indicated by the asymptotes. The system has  $+90$  (deg) phase at low frequencies and transitions to  $-90$  (deg) as the corner frequency is passed.



Corner Frequency:  $\omega = 5$  (rad/sec)

@  $\omega = 1$  (rad/sec):  $M(\omega)|_{\omega=1} = 20\log(100) + 20\log(1) - 40\log(5) = 12$  (dB)

$\omega \ll 5$  (rad/sec):

$M(\omega)$  has a slope of  $+20$  dB/decade due to the zero at the origin.

$\phi = 90$  (deg)

High frequency asymptote:  $\omega \gg 20$  (rad/sec)

$M(\omega)$  has a  $-20$  dB/decade slope

$\phi = -90$  (deg)

**MATLAB® Results:**

For comparison with the above results, the following diagrams were generated for each of the example systems using MATLAB®.

