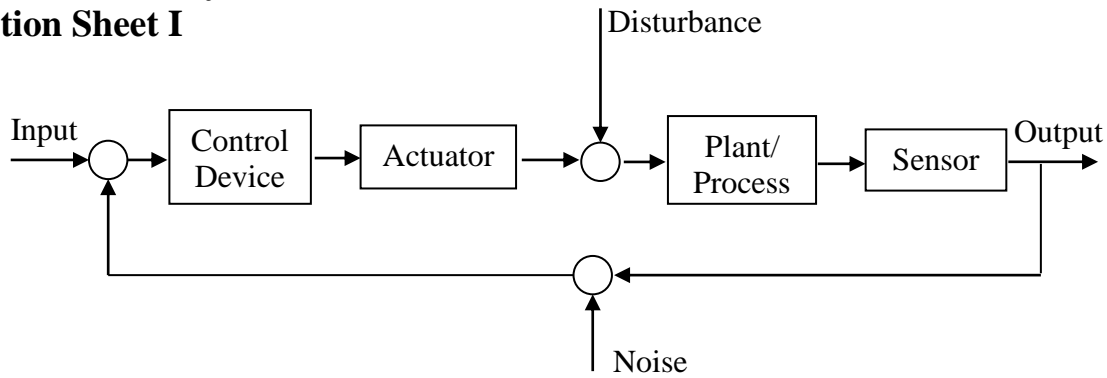
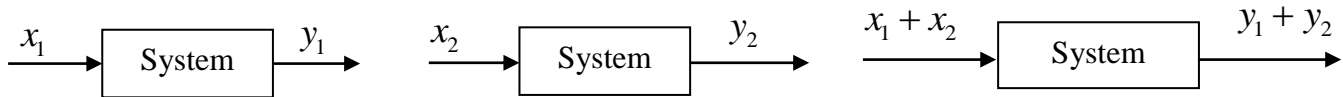


**ME 360 Control Systems**  
**Equation Sheet I**

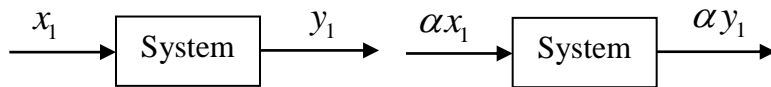


**General Quadratic Equation:**  $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$     **Roots:**  $s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$

**Principle of Superposition:**



**Principle of Homogeneity:** ( $\alpha$  is a scalar)



**Linear Model:**  $\Delta f = m \Delta x$  and  $m = \left. \frac{df}{dx} \right|_{x=x_{eq}}$

**Properties of the Laplace Transform and Inverse Laplace Transforms:**

- $\mathcal{L}(kf(t)) = k\mathcal{L}(f(t)) = kF(s)$     and     $\mathcal{L}^{-1}(kF(s)) = kf(t)$
- $\mathcal{L}(f_1(t) \pm f_2(t)) = F_1(s) \pm F_2(s)$     and     $\mathcal{L}^{-1}(F_1(s) \pm F_2(s)) = f_1(t) \pm f_2(t)$
- $\mathcal{L}\left(\frac{df}{dt}\right) = sF(s) - f(0)$     and     $\mathcal{L}\left(\frac{d^2f}{dt^2}\right) = s^2F(s) - sf(0) - \frac{df}{dt}(0)$
- Final Value Theorem:  $\lim_{t \rightarrow \infty} (f(t)) = \lim_{s \rightarrow 0} (sF(s))$     (if the time limit exists)

**General Partial Fraction Expansion:**

$$X(s) = \frac{K_1}{(s + s_1)} + \frac{K_2}{(s + s_2)} + \dots + \frac{K_N}{(s + s_N)} +$$

$$\frac{A_1s + B_1}{(s^2 + 2\zeta\omega_n s + \omega_n^2)_1} + \frac{A_2s + B_2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)_2} + \dots + \frac{A_Ms + B_M}{(s^2 + 2\zeta\omega_n s + \omega_n^2)_M}$$

$K_i = \left[ (s + s_i) X(s) \right]_{s=-s_i}$  and the  $A$  and  $B$  coefficients are found by clearing fractions.